

Statik mit finiten Elementen

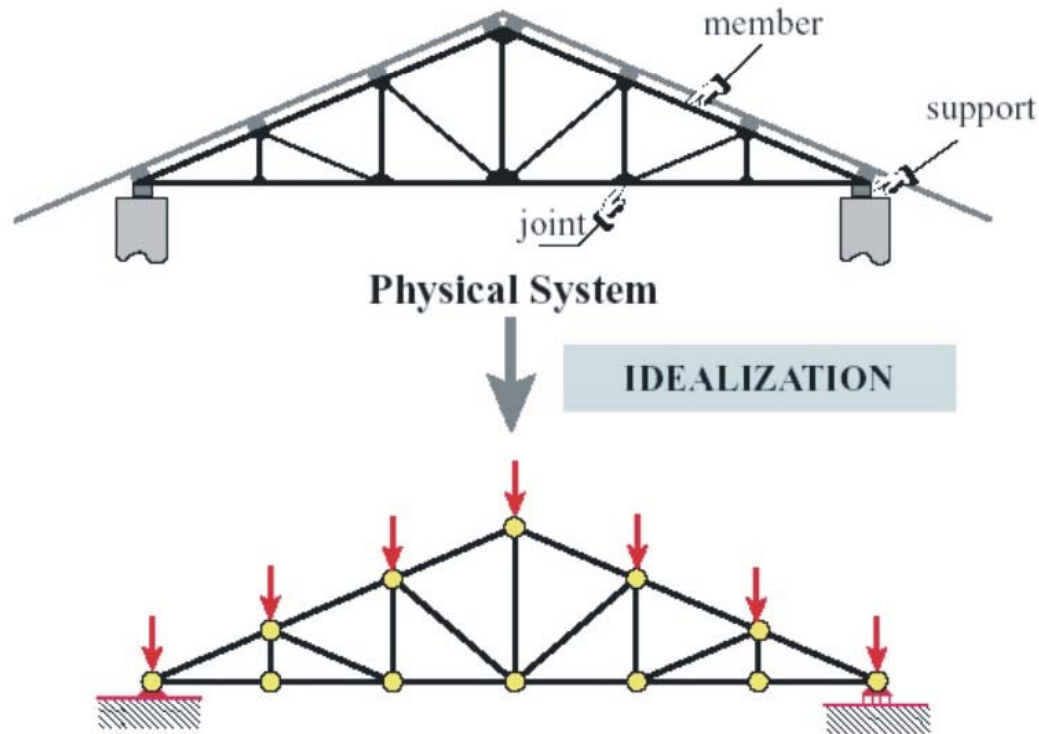
Fachgebiet Baustatik

Prof. Dr.-Ing. Friedel Hartmann

Was sind finite Elemente?

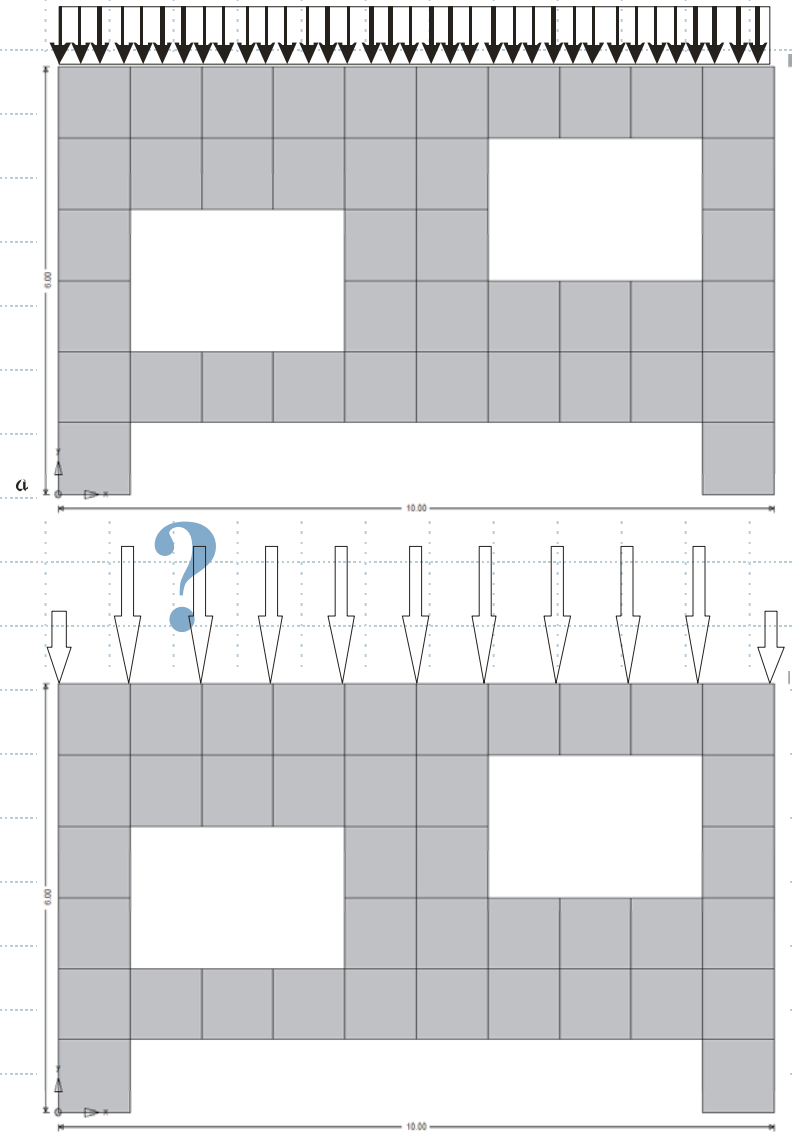
- ◆ Knotenkräfte gibt es die ?
- ◆ Der äquivalente Lastfall
- ◆ Einflussfunktionen
- ◆ Folgerungen für die Praxis

Modellieren mit finiten Elementen

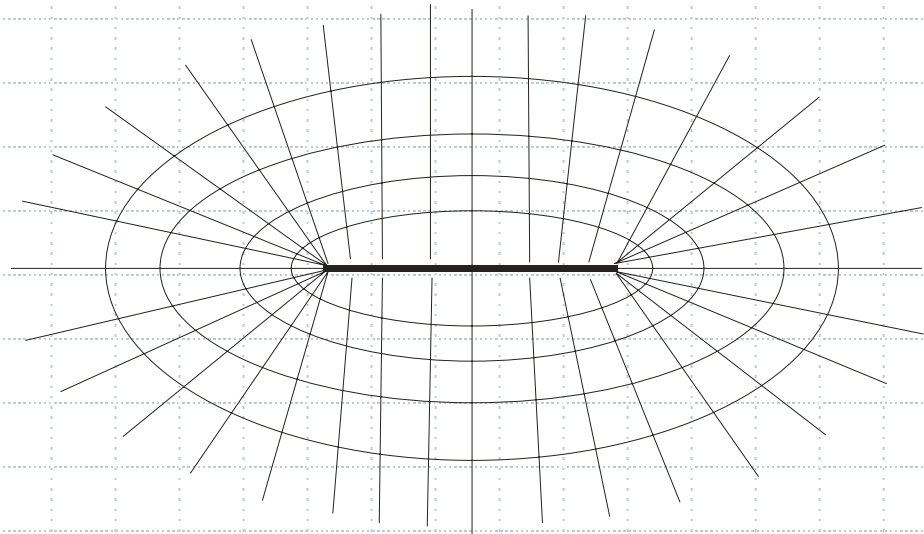
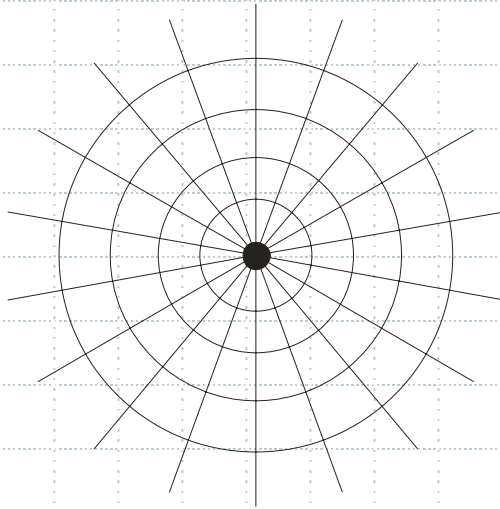
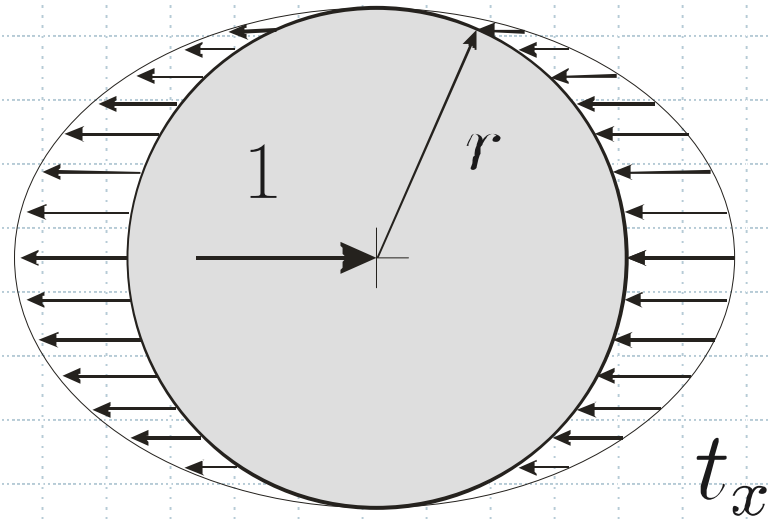


source: University of Colorado

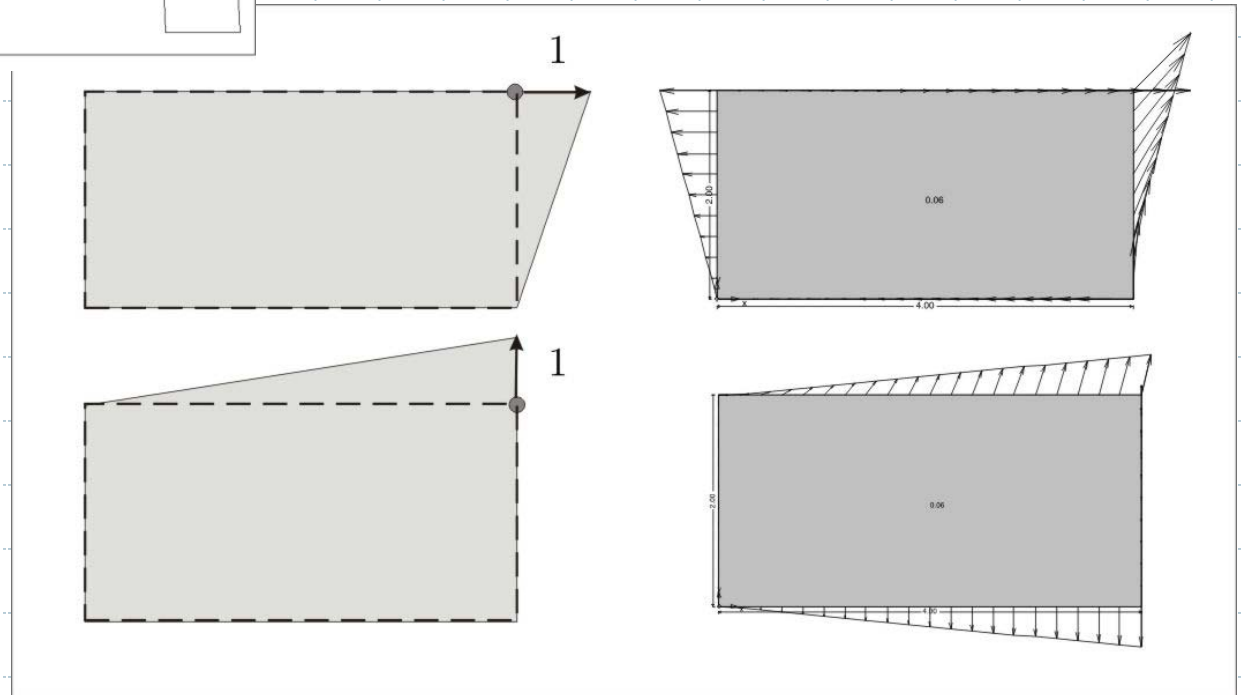
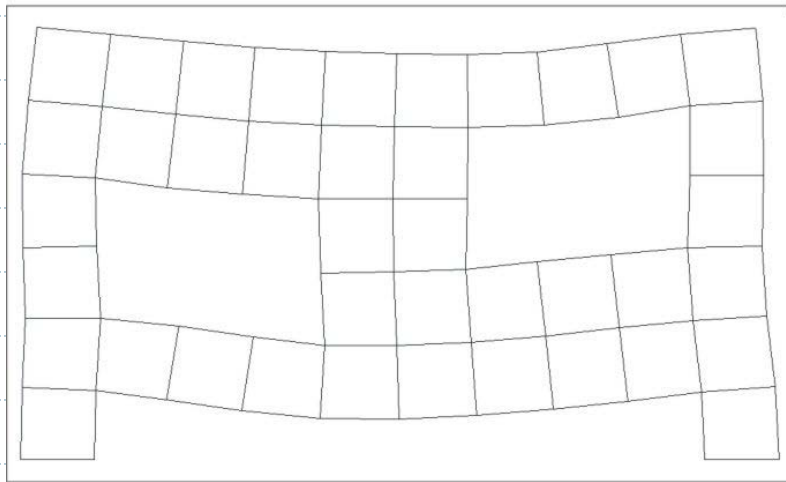
Das FE Modell



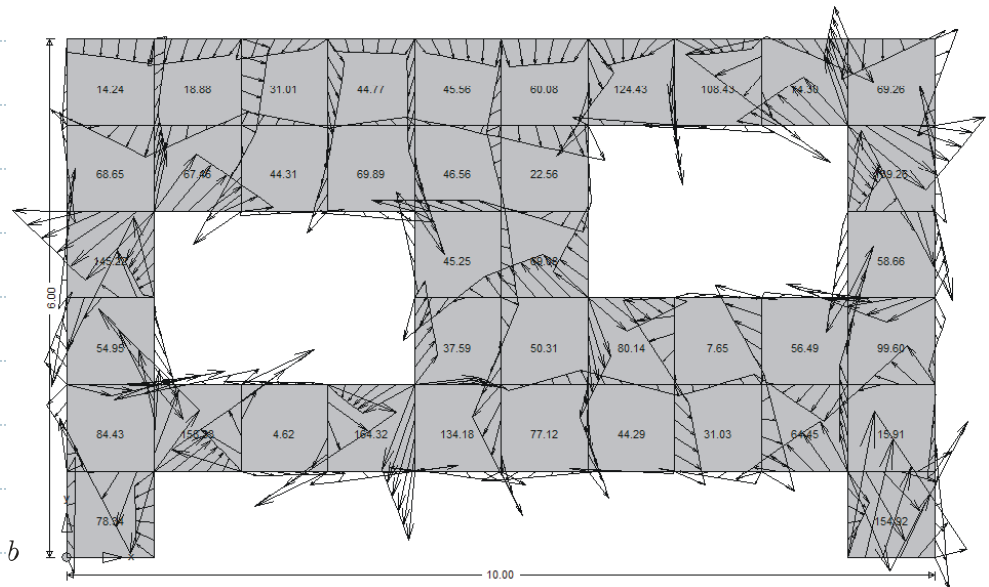
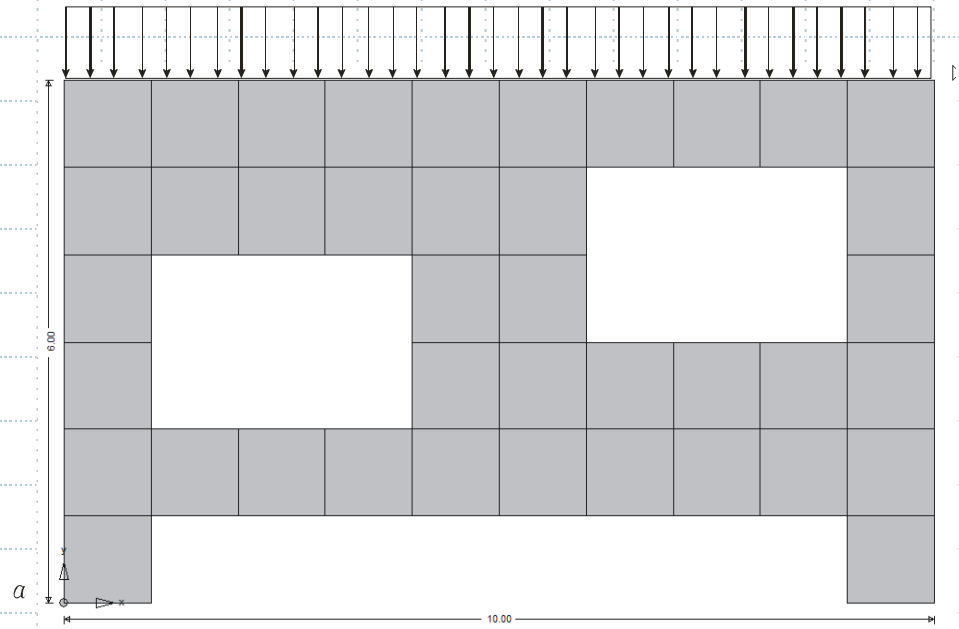
Einzelkräfte



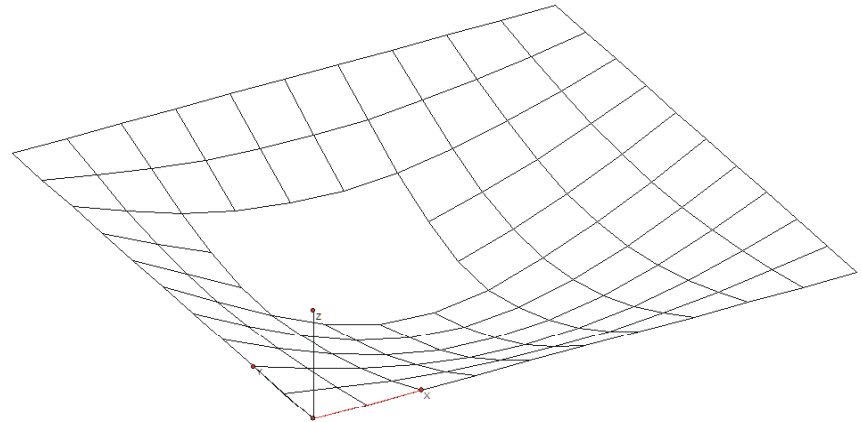
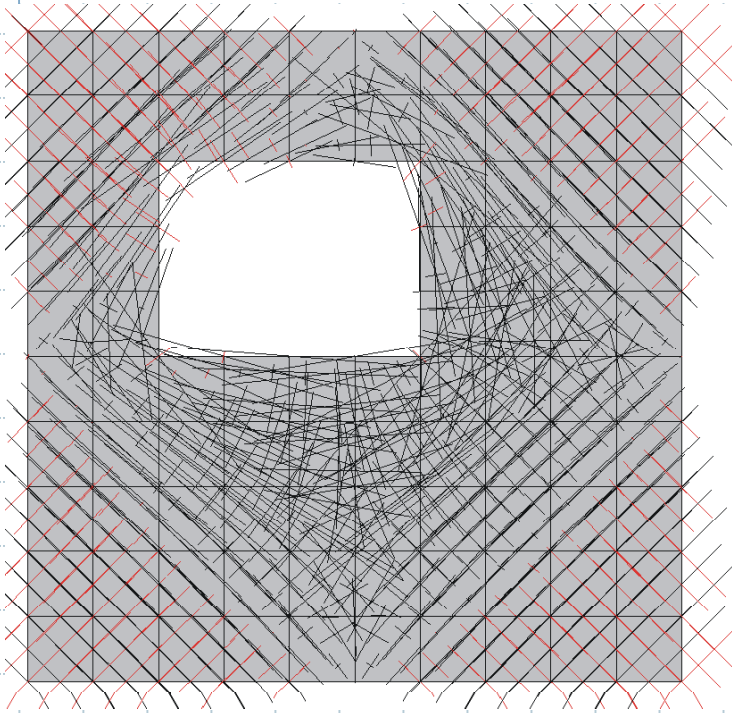
Verformungen -> Kräfte



Der FE-Lastfall



Platte LF g



Der Lastfall g_h



6.50	6.50	6.50	6.50	6.50	6.50	6.50	6.50	6.50	6.50
6.50	6.50	6.50	6.50	6.50	6.50	6.50	6.50	6.50	6.50
6.50	6.50					6.50	6.50	6.50	6.50
6.50	6.50					6.50	6.50	6.50	6.50
6.50	6.50					6.50	6.50	6.50	6.50
6.50	6.50	6.50	6.50	6.50	6.50	6.50	6.50	6.50	6.50
6.50	6.50	6.50	6.50	6.50	6.50	6.50	6.50	6.50	6.50
6.50	6.50	6.50	6.50	6.50	6.50	6.50	6.50	6.50	6.50
6.50	6.50	6.50	6.50	6.50	6.50	6.50	6.50	6.50	6.50
6.50	6.50	6.50	6.50	6.50	6.50	6.50	6.50	6.50	6.50

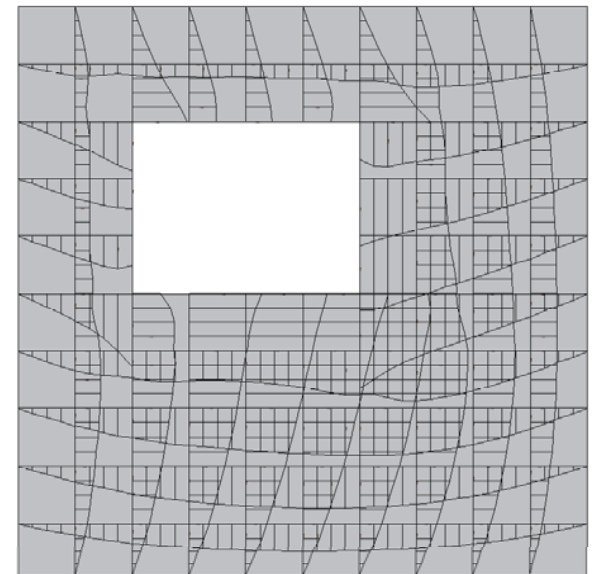
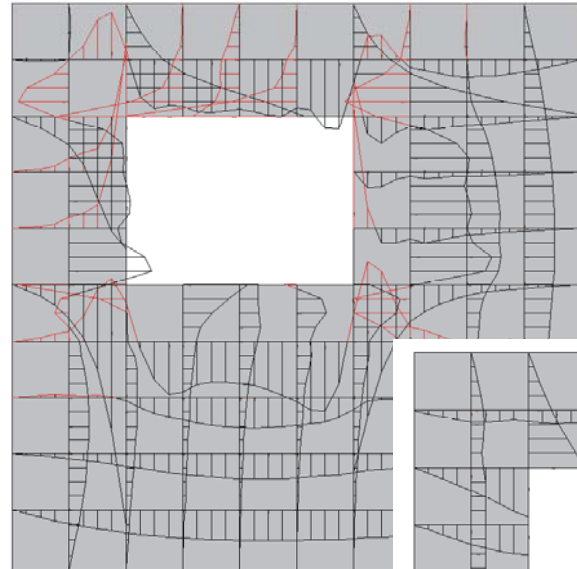
3.98	3.35	-2.55	0.53	0.10	-3
3.54	42.36	-29.62	-3.47	-4.77	-3

-2.21	-29.15
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0.51	-7.29
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-4.18	-38.20
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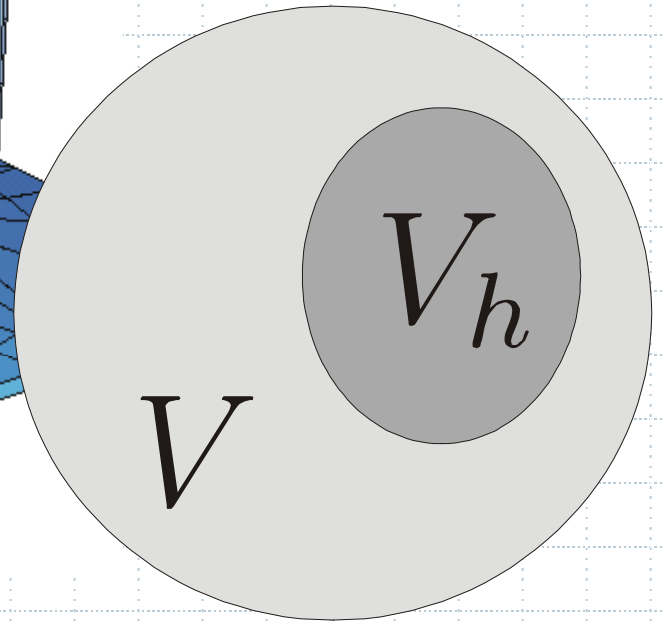
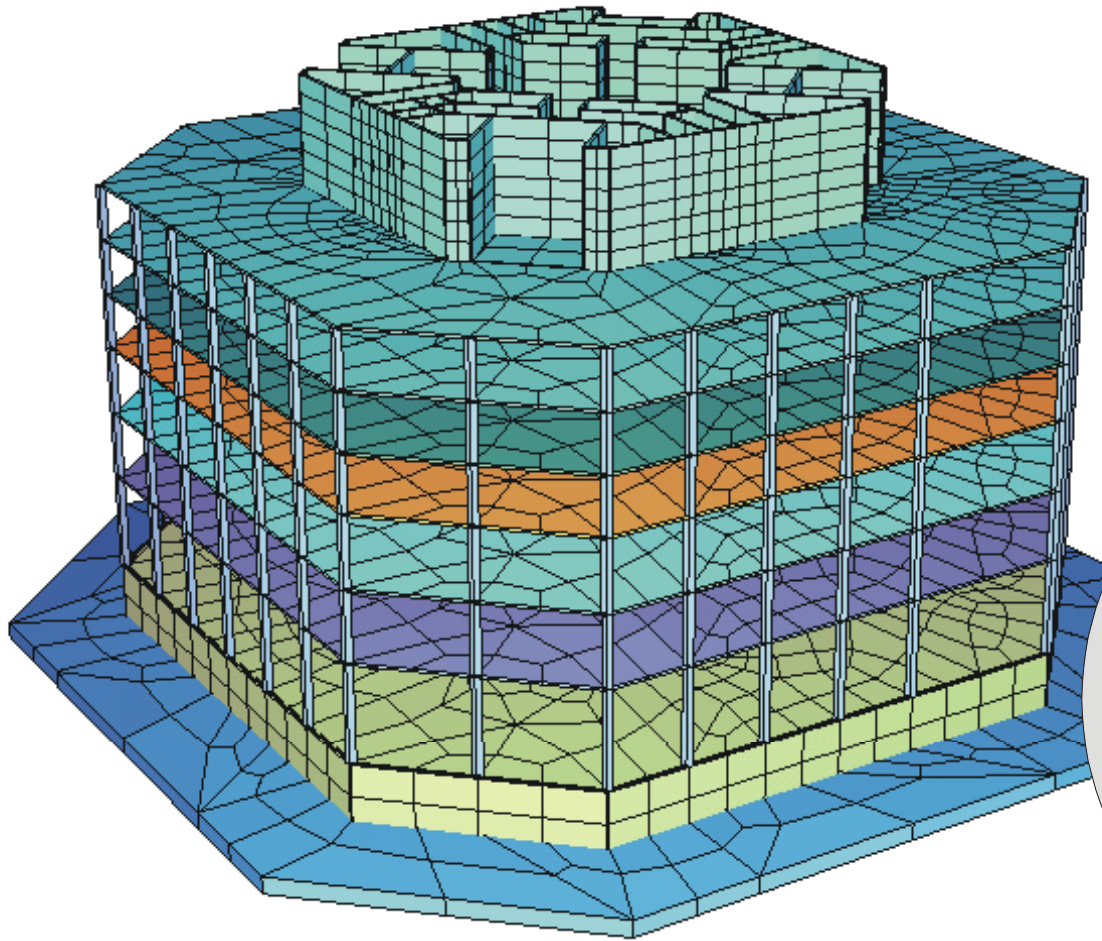
						-55.25	-5.75	-0.02	0.30
1.99	39.95	-22.53	-3.74	-6.62	-48.01	62.59	6.19	1.84	0.60
3.28	7.03	1.17	-1.22	-1.91	-2.45	8.62	7.96	3.62	1.09
1.57	3.13	2.47	1.39	1.02	1.58	3.59	4.58	3.40	1.27
1.36	2.45	2.14	1.65	1.42	1.61	2.22	2.83	2.82	1.43
1.58	1.25	0.90	0.69	0.60	0.64	0.81	1.06	1.38	1.62



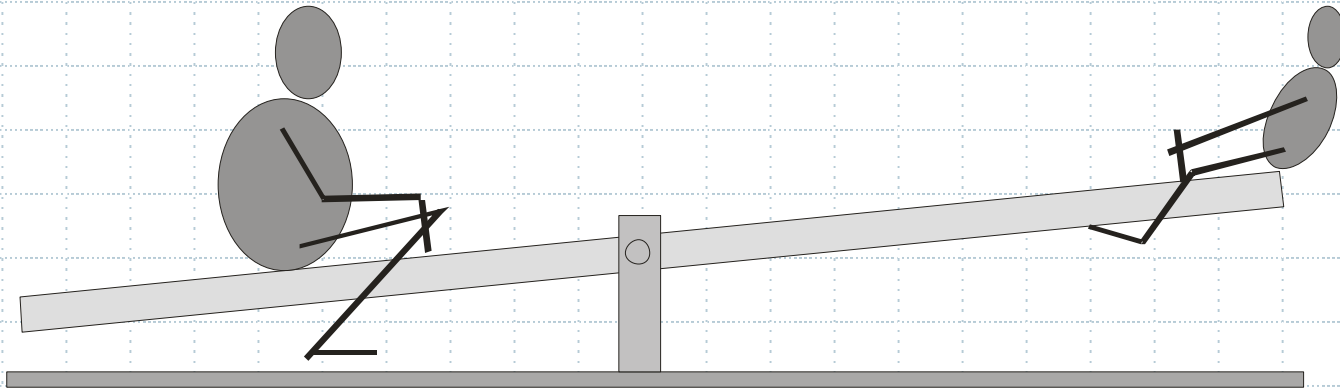
Frage

◆ Wie wählt ein FE-Programm den Ersatzlastfall aus ?

Finite Elemente = Restriktion

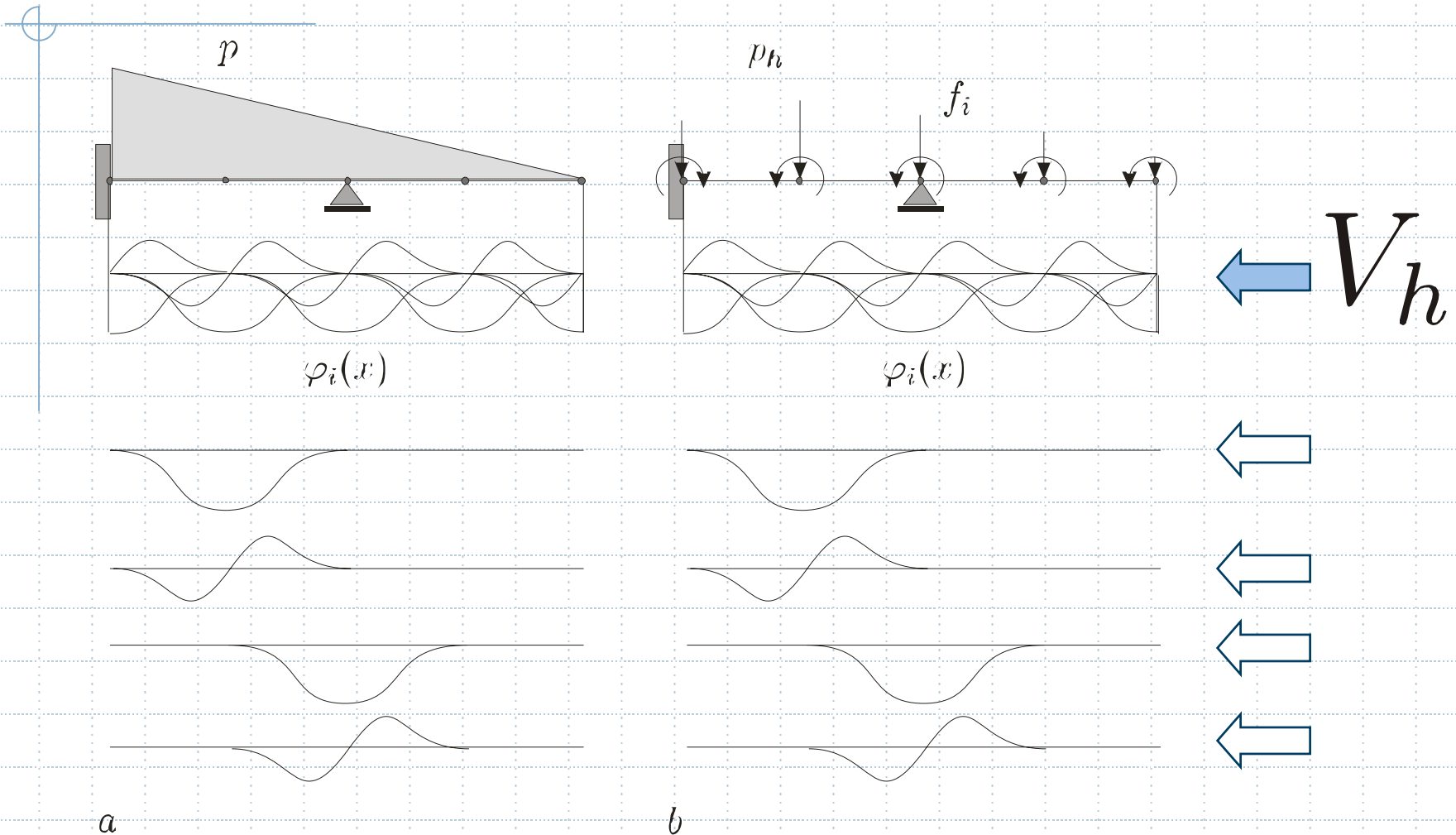


Äquivalenz

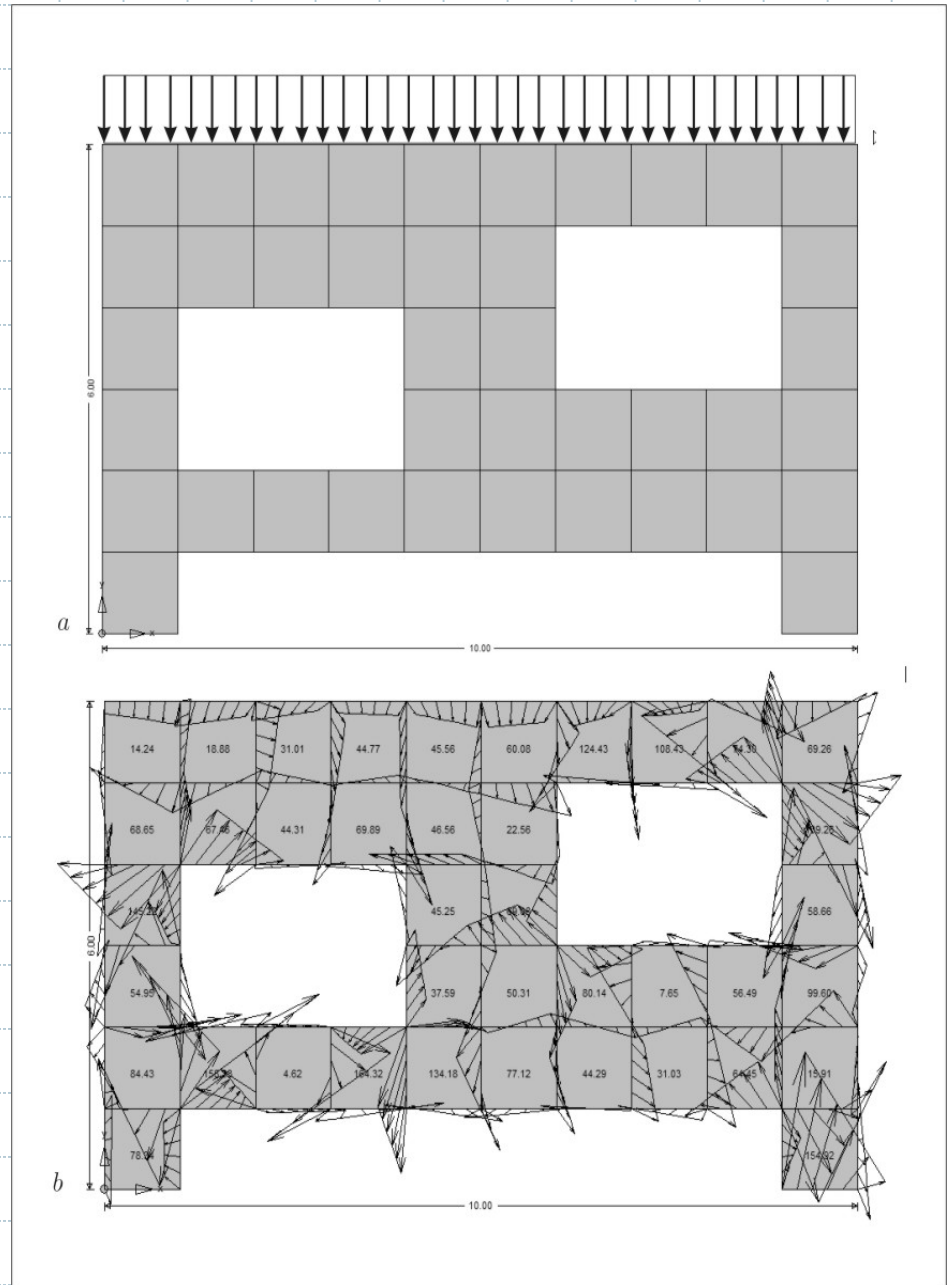
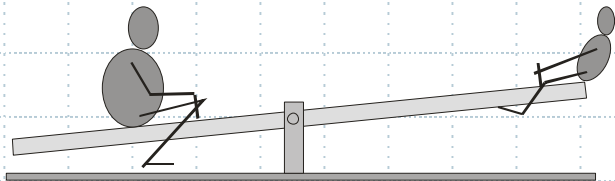
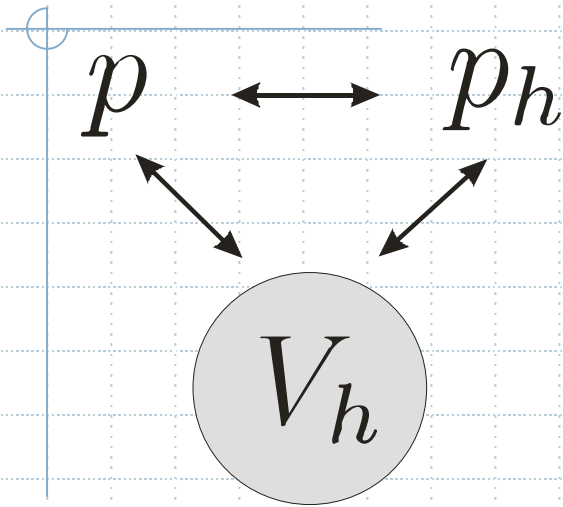


Vater und Sohn leisten bei jeder Drehung die gleiche Arbeit.

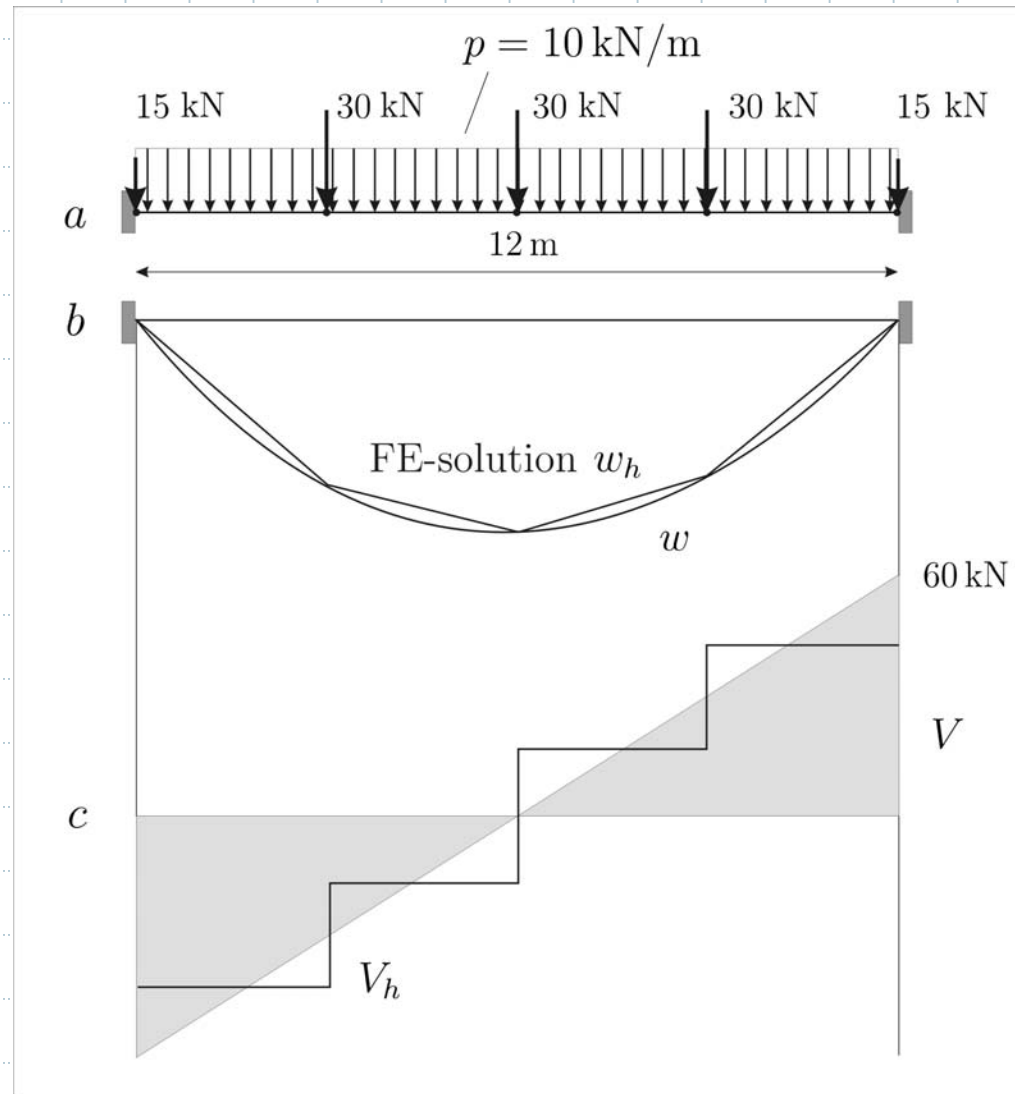
Gleiche Arbeit auf gleichen Wegen



Der FE Lastfall

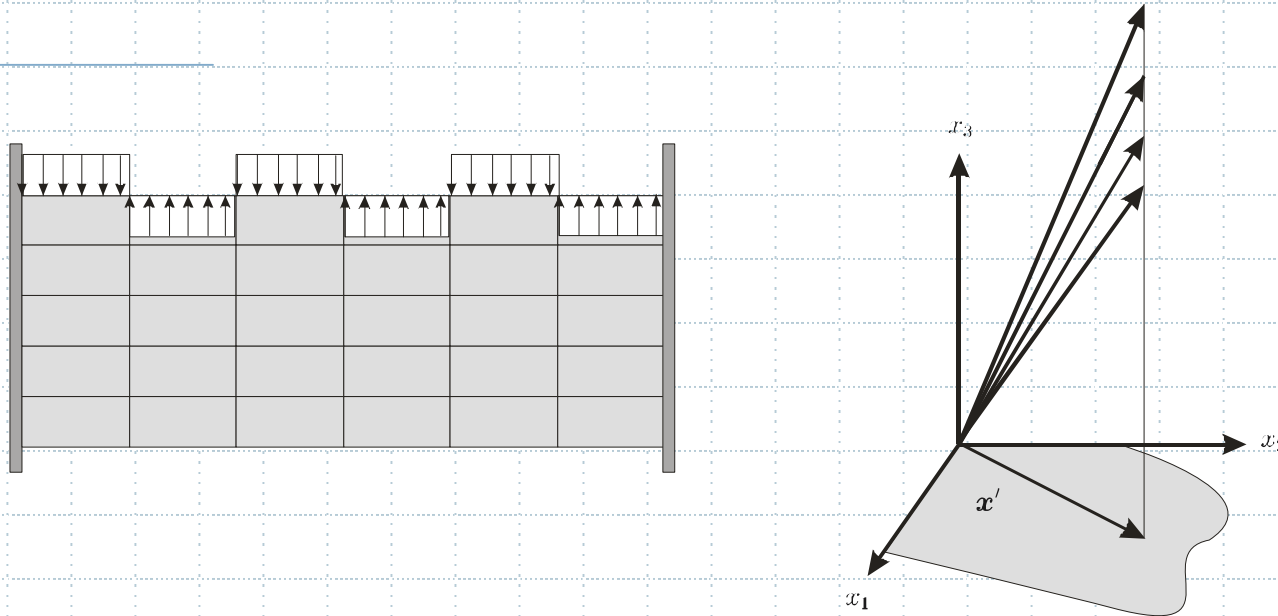


FE-Methode



Ein möglichst kleiner Fehler in den Spannungen ist das Ziel

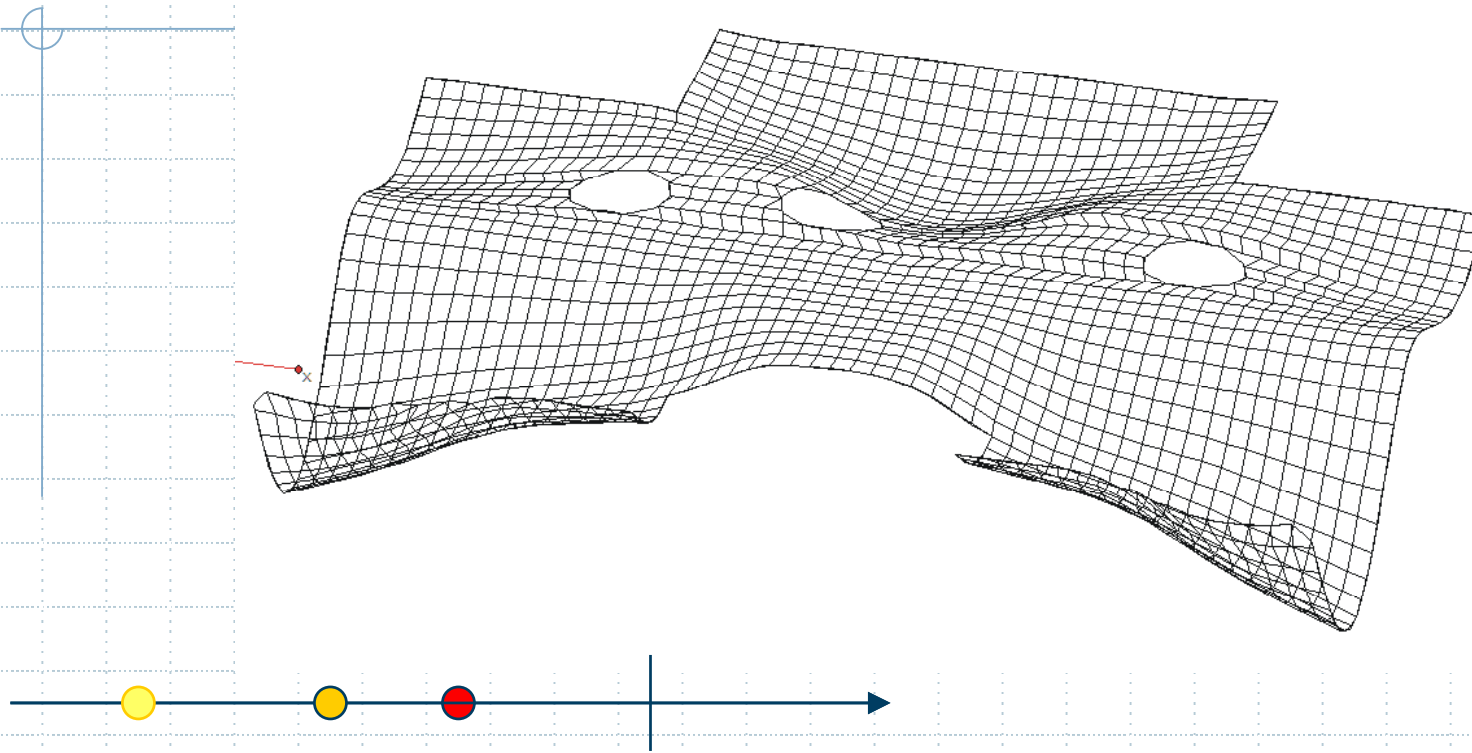
FE = Projektionsverfahren



Ziel: Minimaler Fehler in den Spannungen

Manche LF kann man mit finiten Elementen nicht lösen
Unterschiedliche LF haben dieselbe FE-Lösung

Interpolation ? – Nein, aber



Ein Netz ist dann gut, wenn man die Biegefläche auf ihm gut interpolieren könnte.

2. Teil

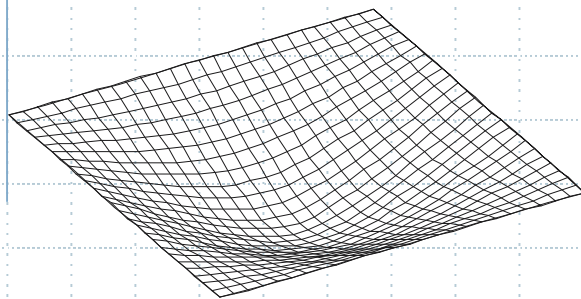
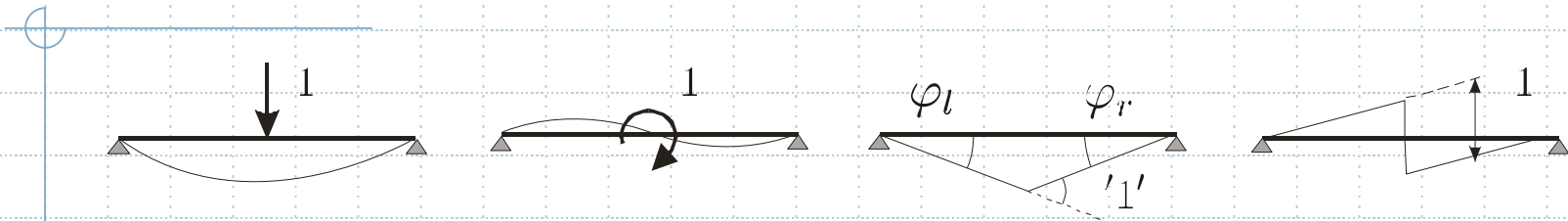
Warum ein FE-Programm falsch rechnet.

Einflussfunktionen

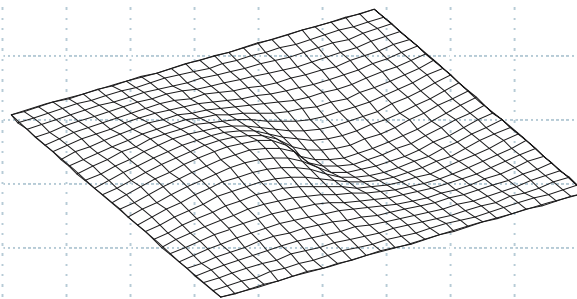


- ◆ Die Einflussfunktionen sind der Schlüssel zur Statik
- ◆ und zu den finiten Elementen.

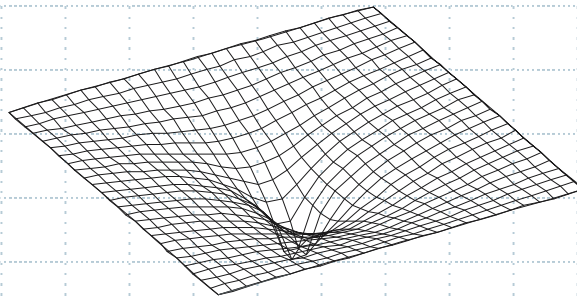
Typen von Einflussfunktionen



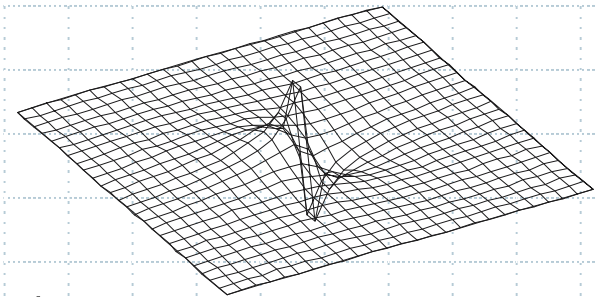
a



b

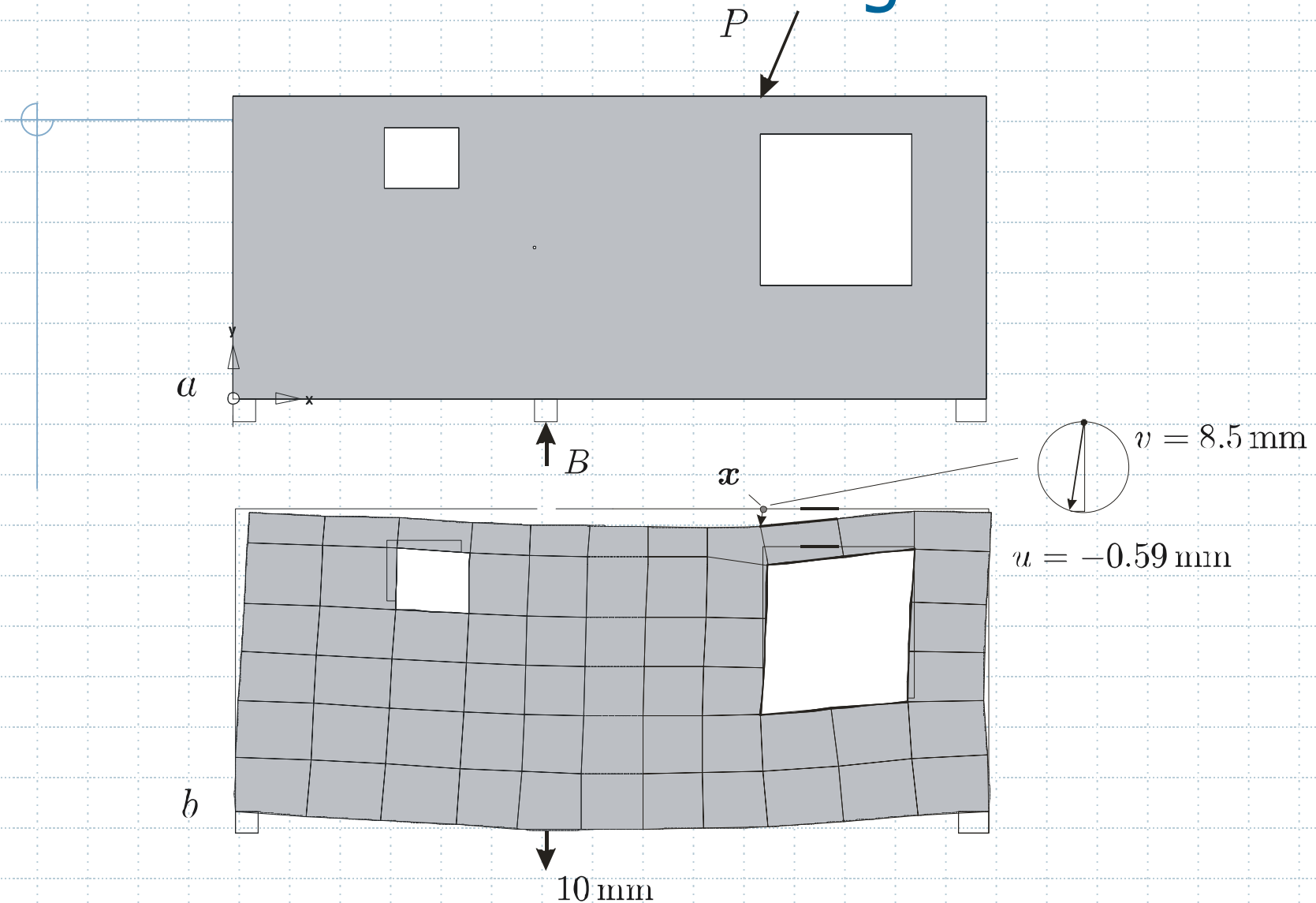


c



d

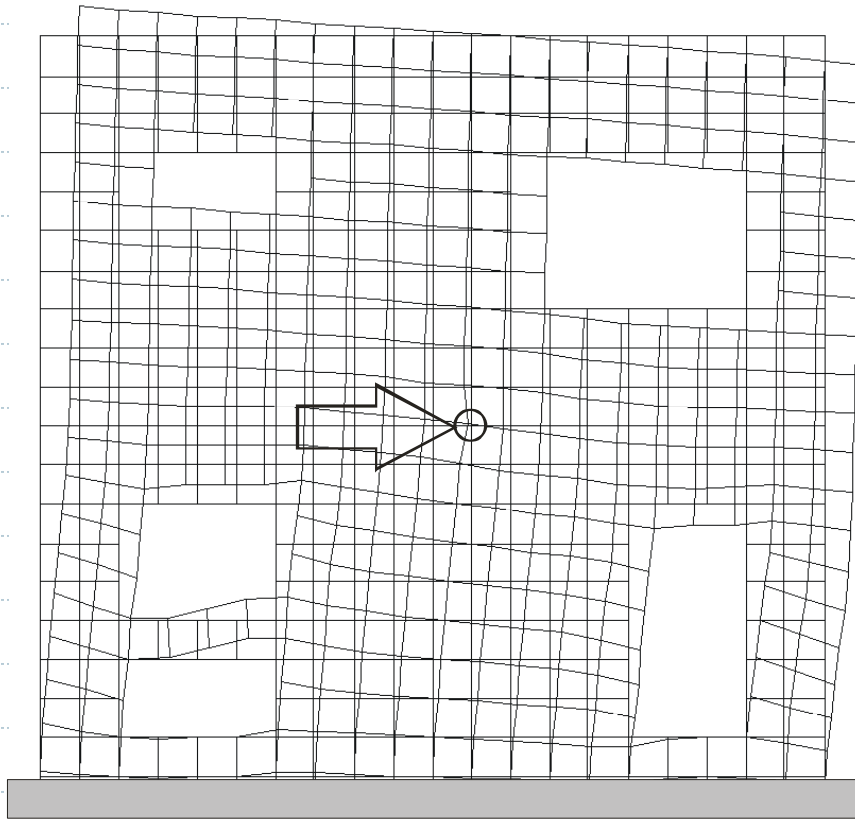
Einflussfunktion für eine Lagerkraft



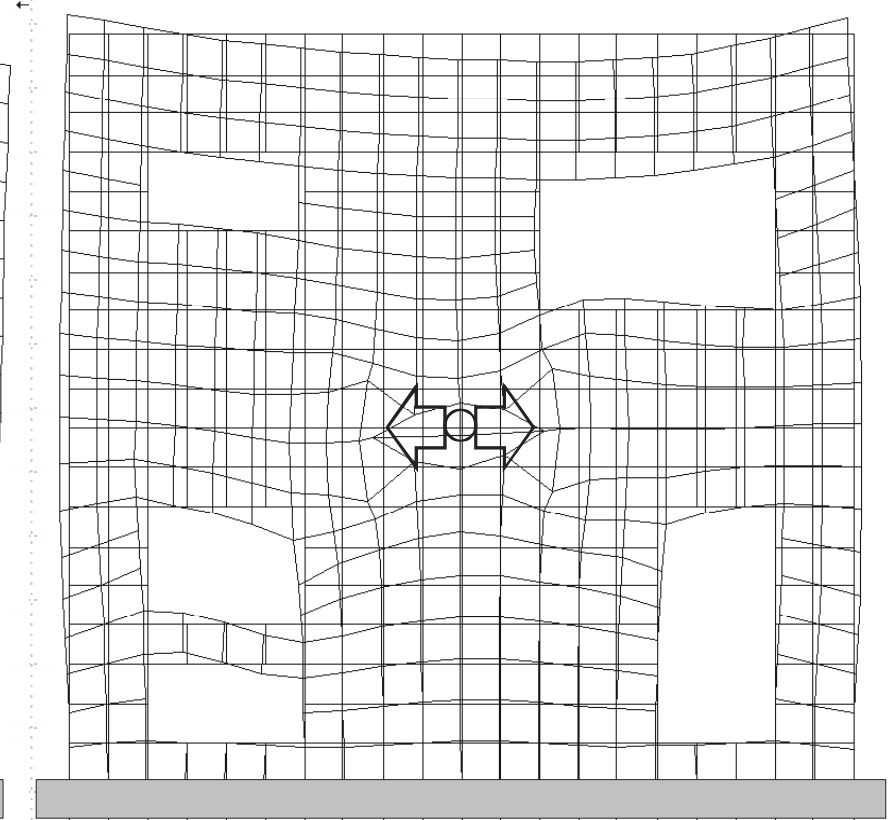
Einflussfunktionen für Punktwerte

point force

dislocation



a

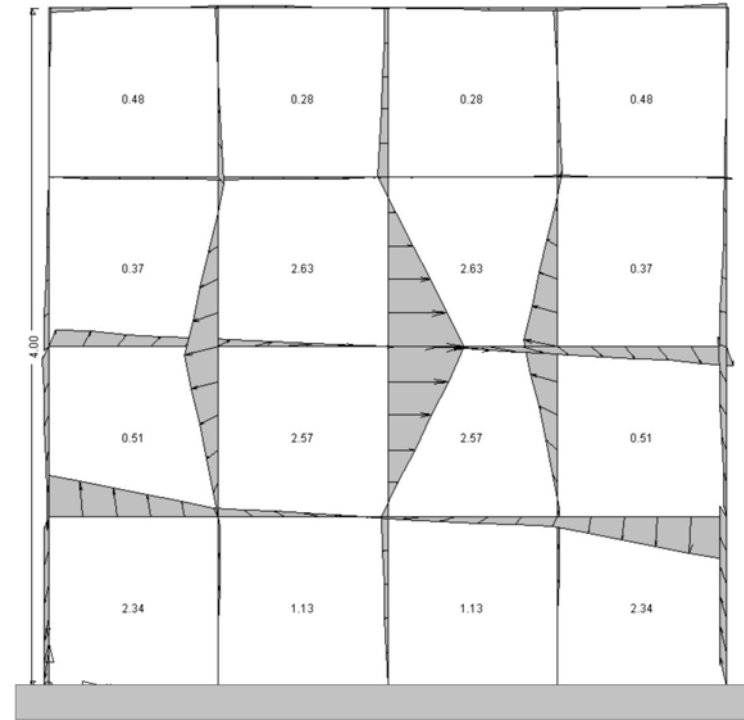
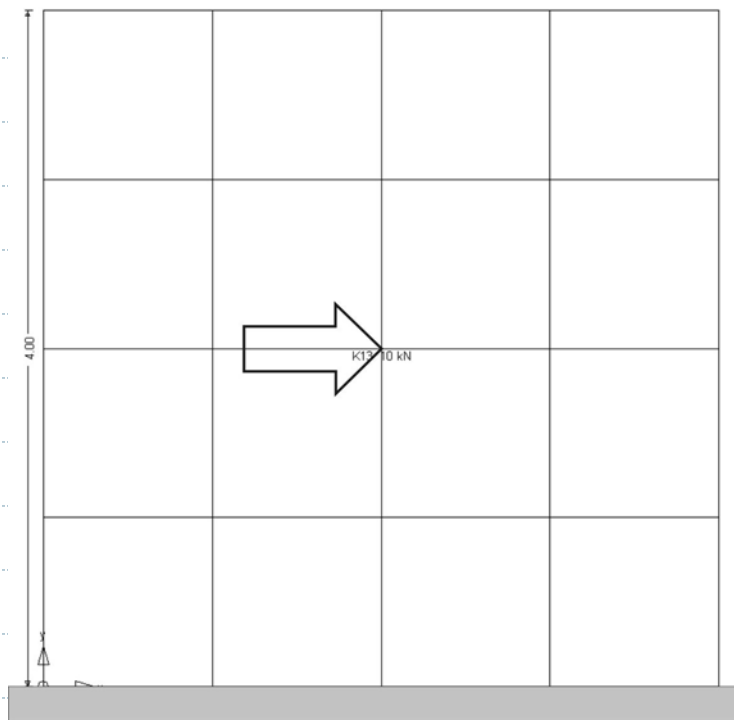


b

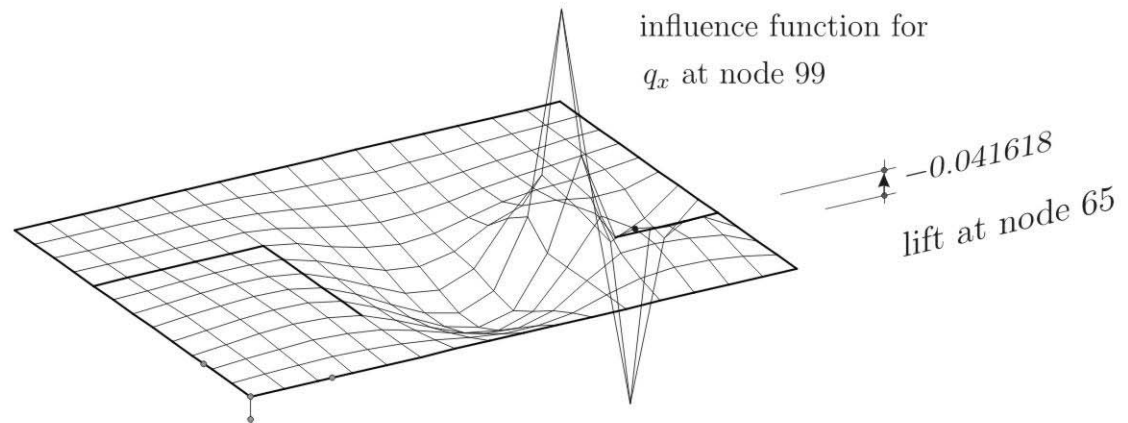
Der zentrale Satz

- ◆ Das FE-Programm berechnet alle Werte mit genäherten Einflussfunktionen
- ◆ Verformungen, Spannungen, Lagerkräfte, Schnittkräfte, alles ...
- ◆ Je feiner das Netz, um so besser können die Einflussfunktionen angenähert werden und um so genauer sind die Ergebnisse.

Näherungen

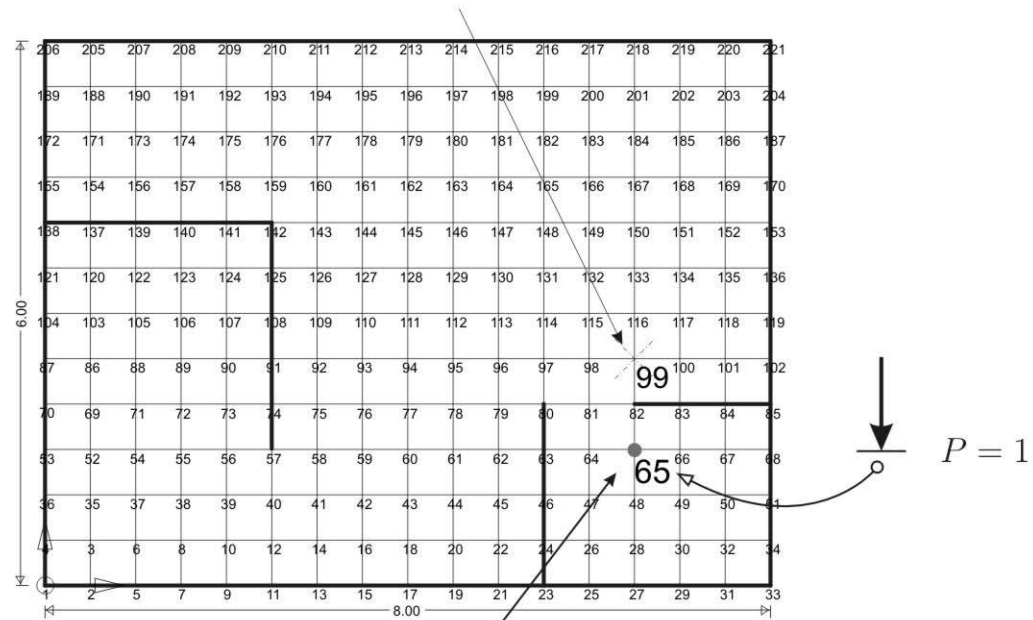


Querkraft



a

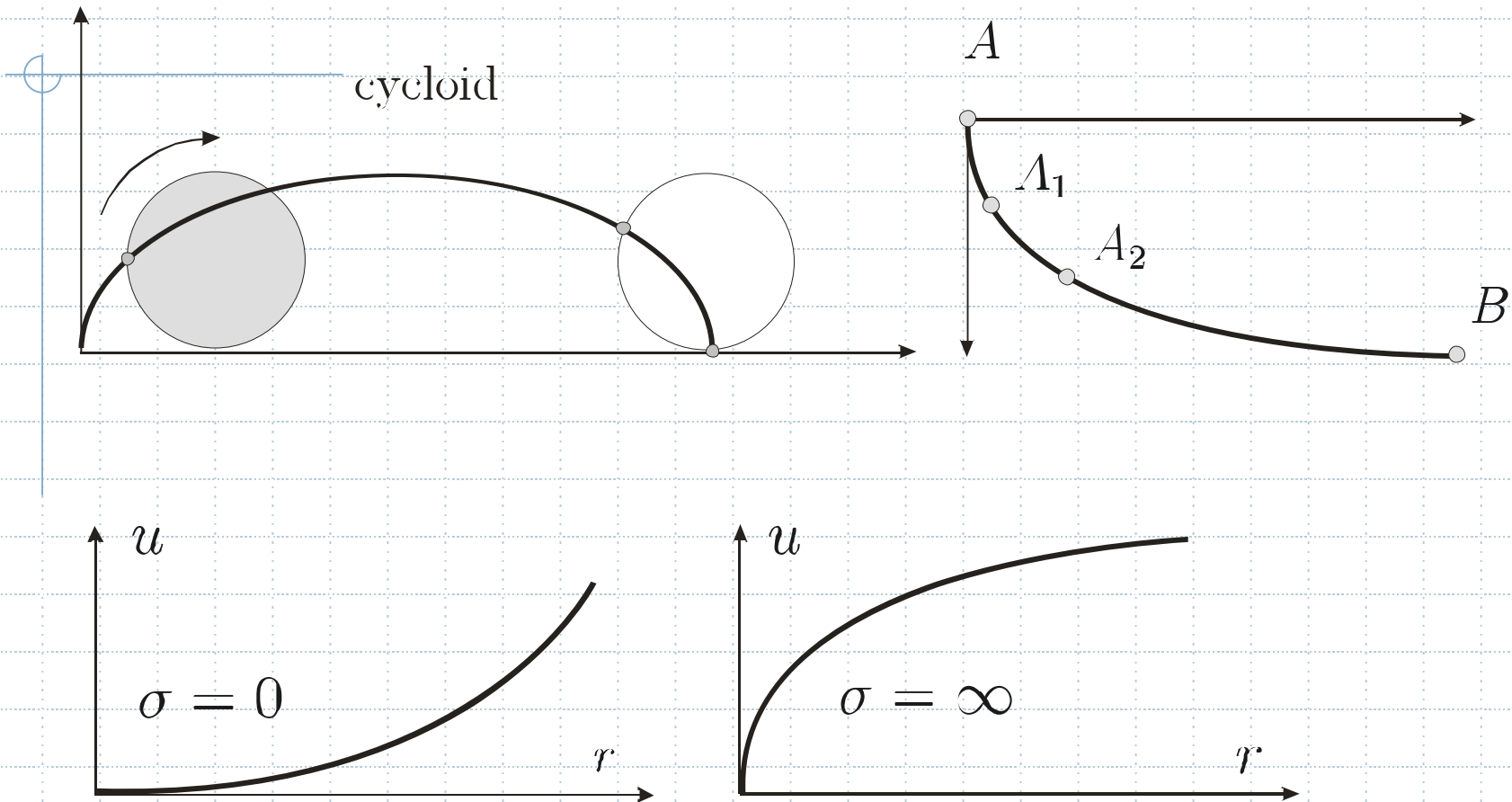
$$q_x = -0.041618 = w \times P = -0.041618 \times 1$$



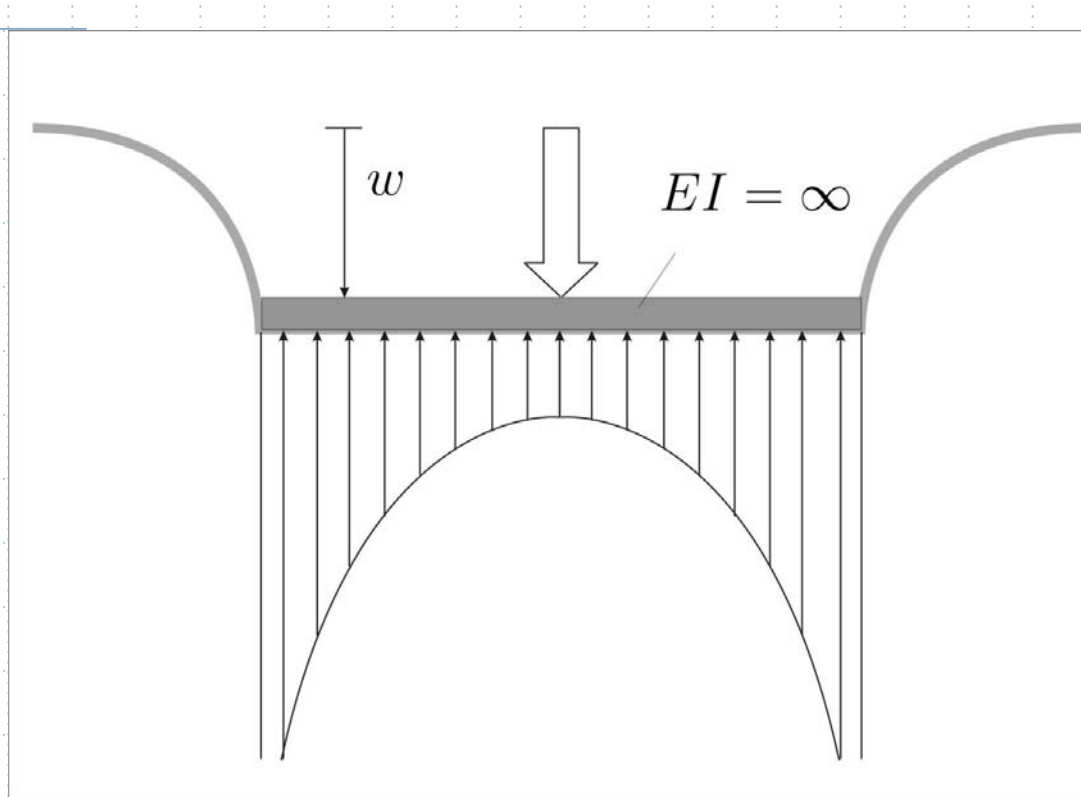
b

$$w = -0.041618 \text{ m}$$

Singularitäten (die 2. Ursache für Fehler)



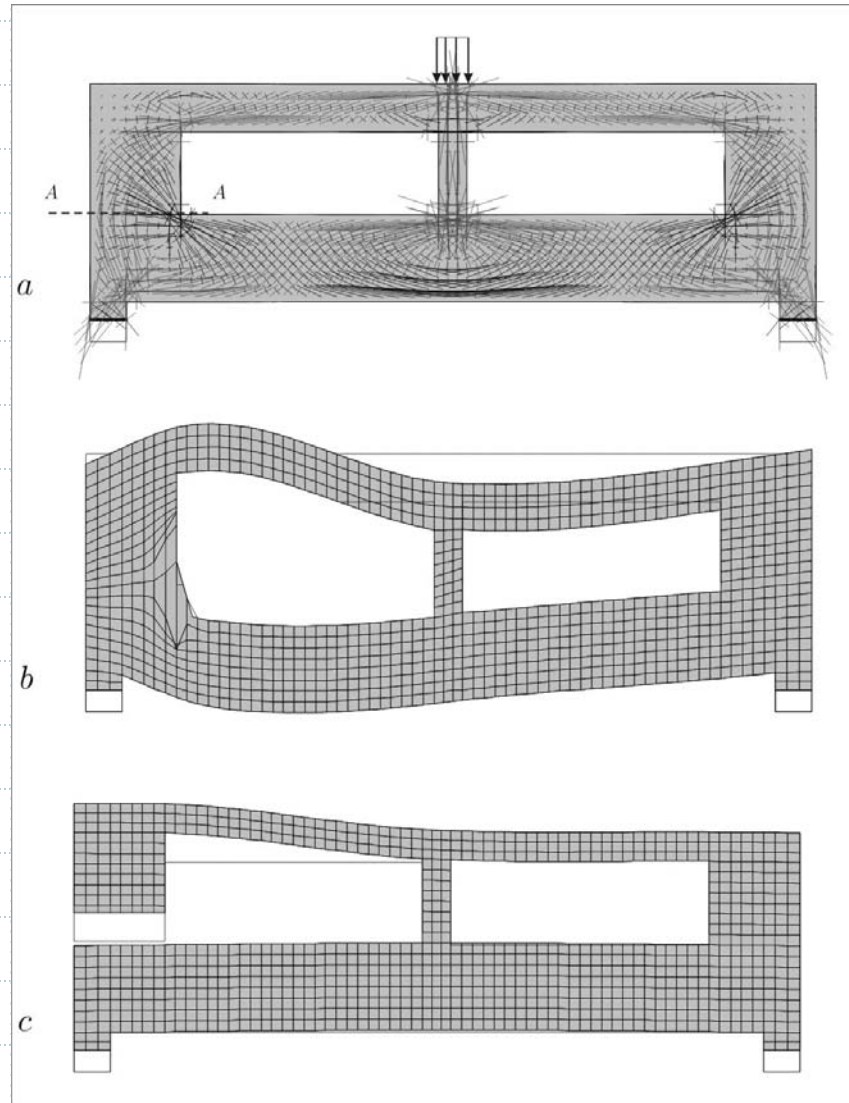
Starrer Stempel



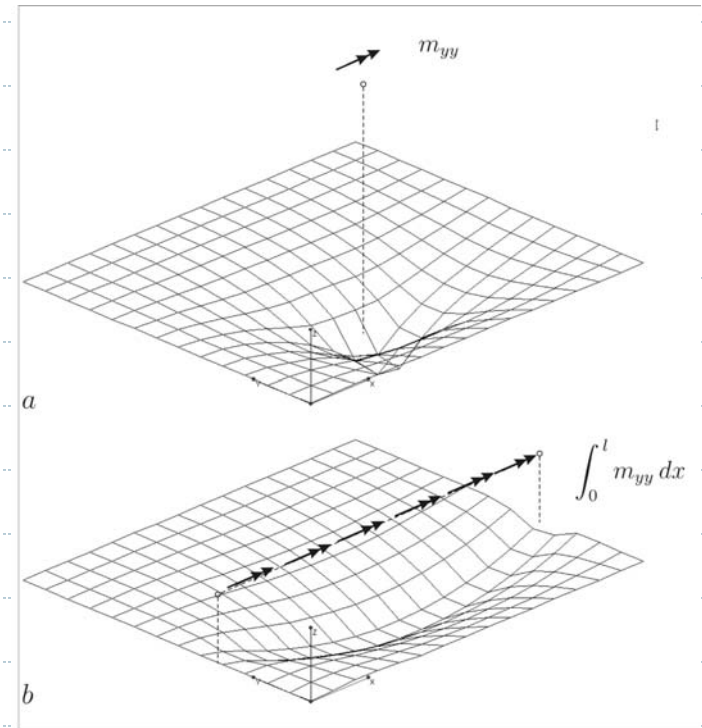
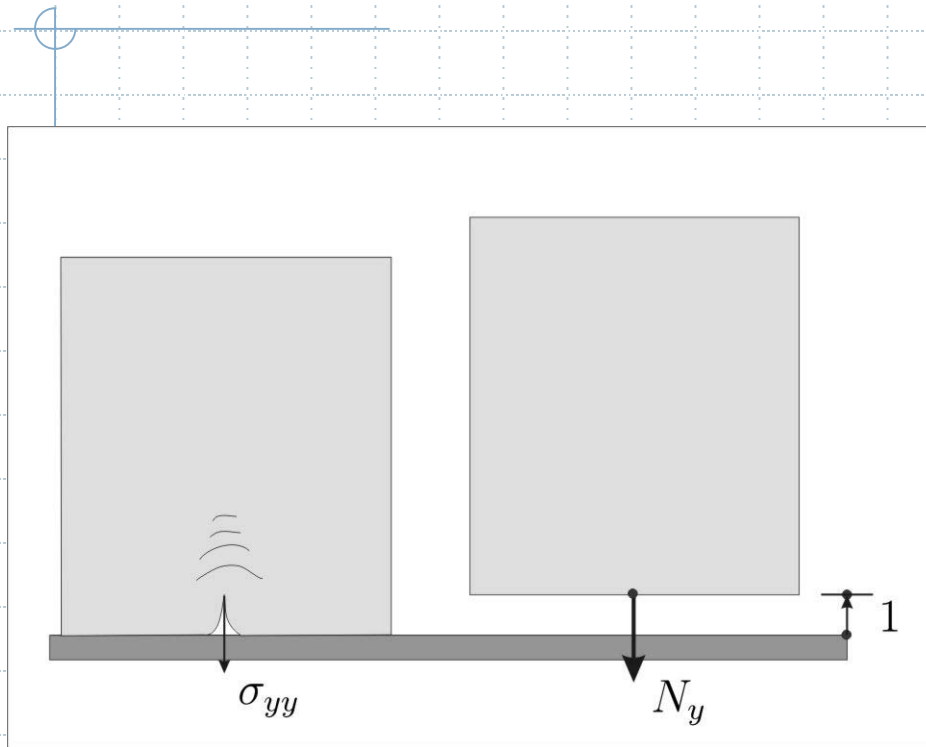
Folgerungen für die Praxis

- Integrale Spannungen sind genauer
- Lagerkräfte sind genau
- Das Gleichgewicht ist verletzt
- Stützenanschnittsmomente sind problematisch

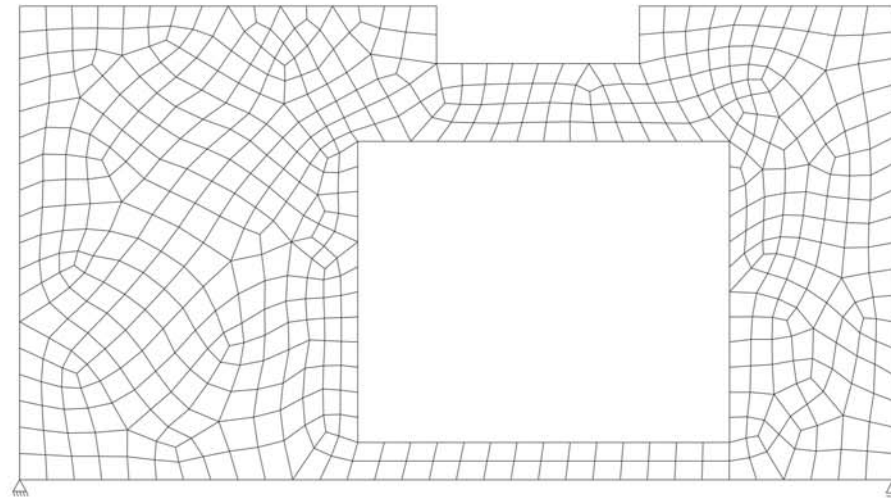
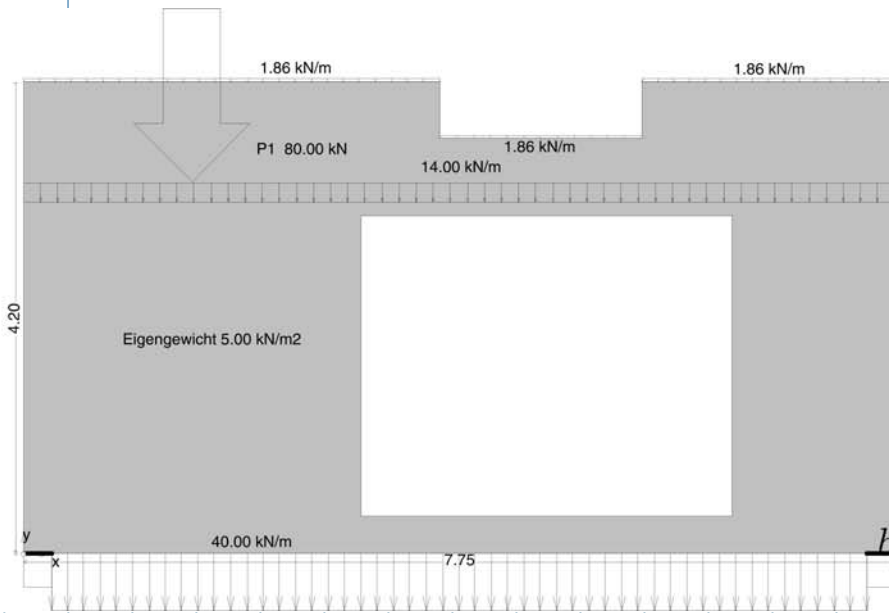
Integrale Spannungen



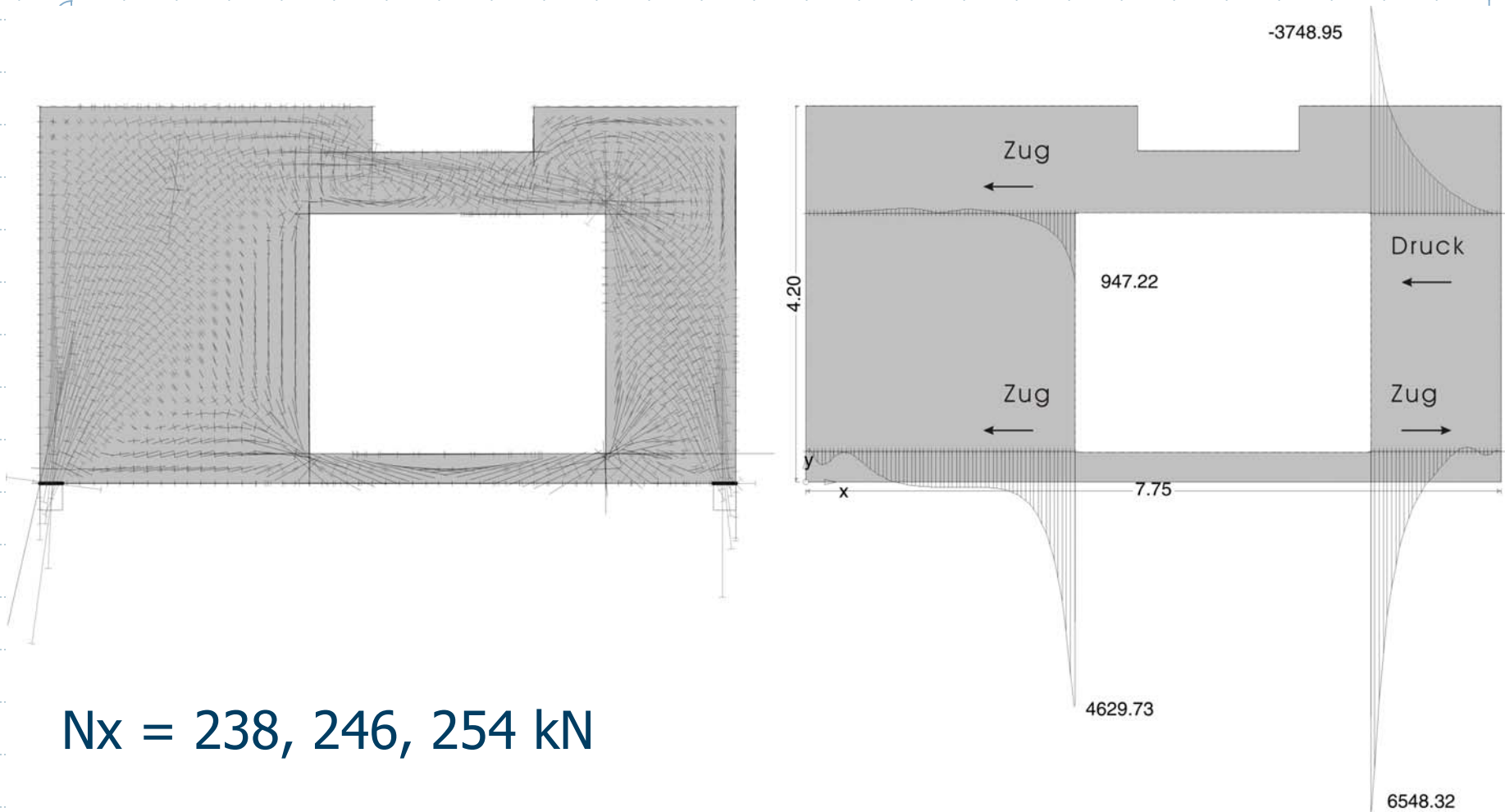
Punkt vs. Schnitt



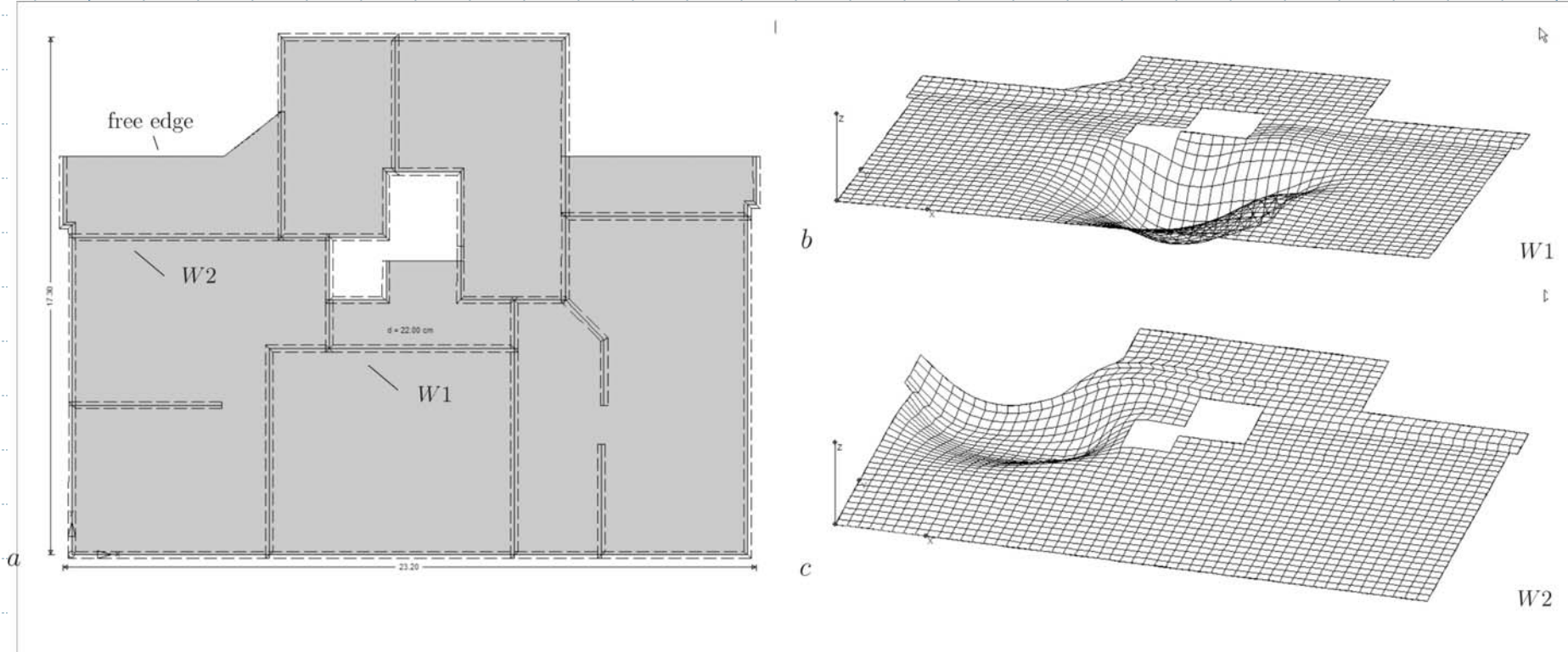
Singuläre Spannungen



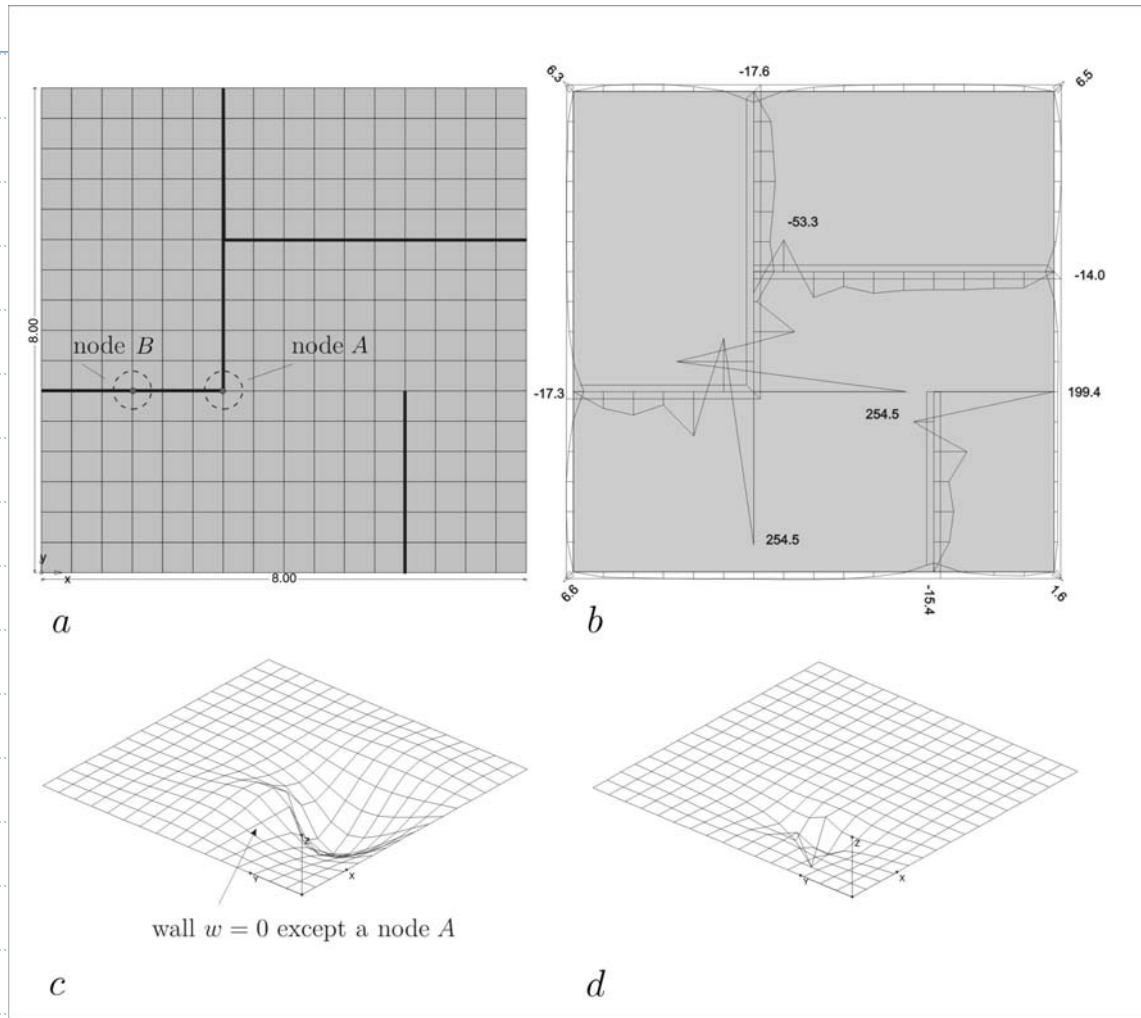
Schnitte statt einzelne Werte



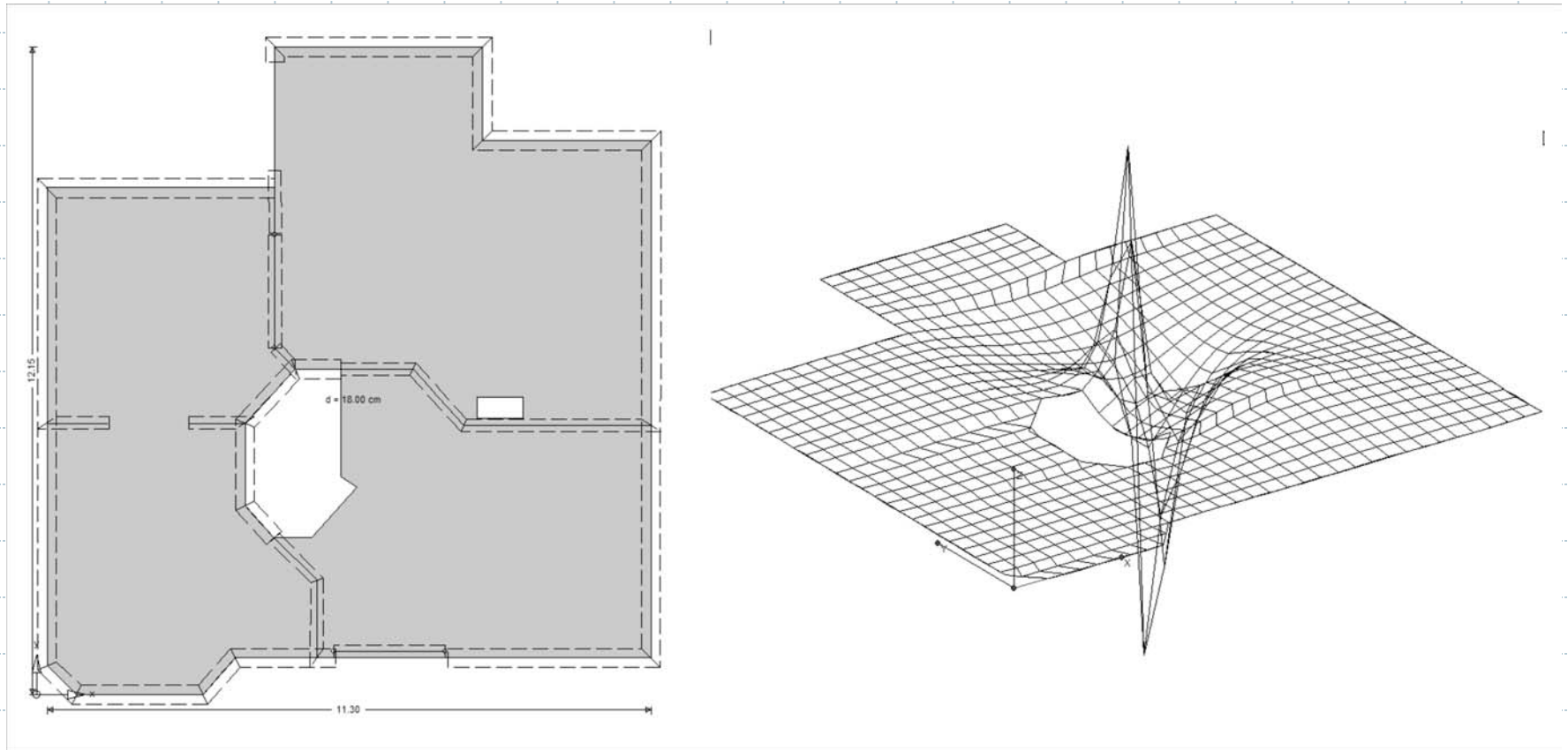
Lagerkräfte sind genau



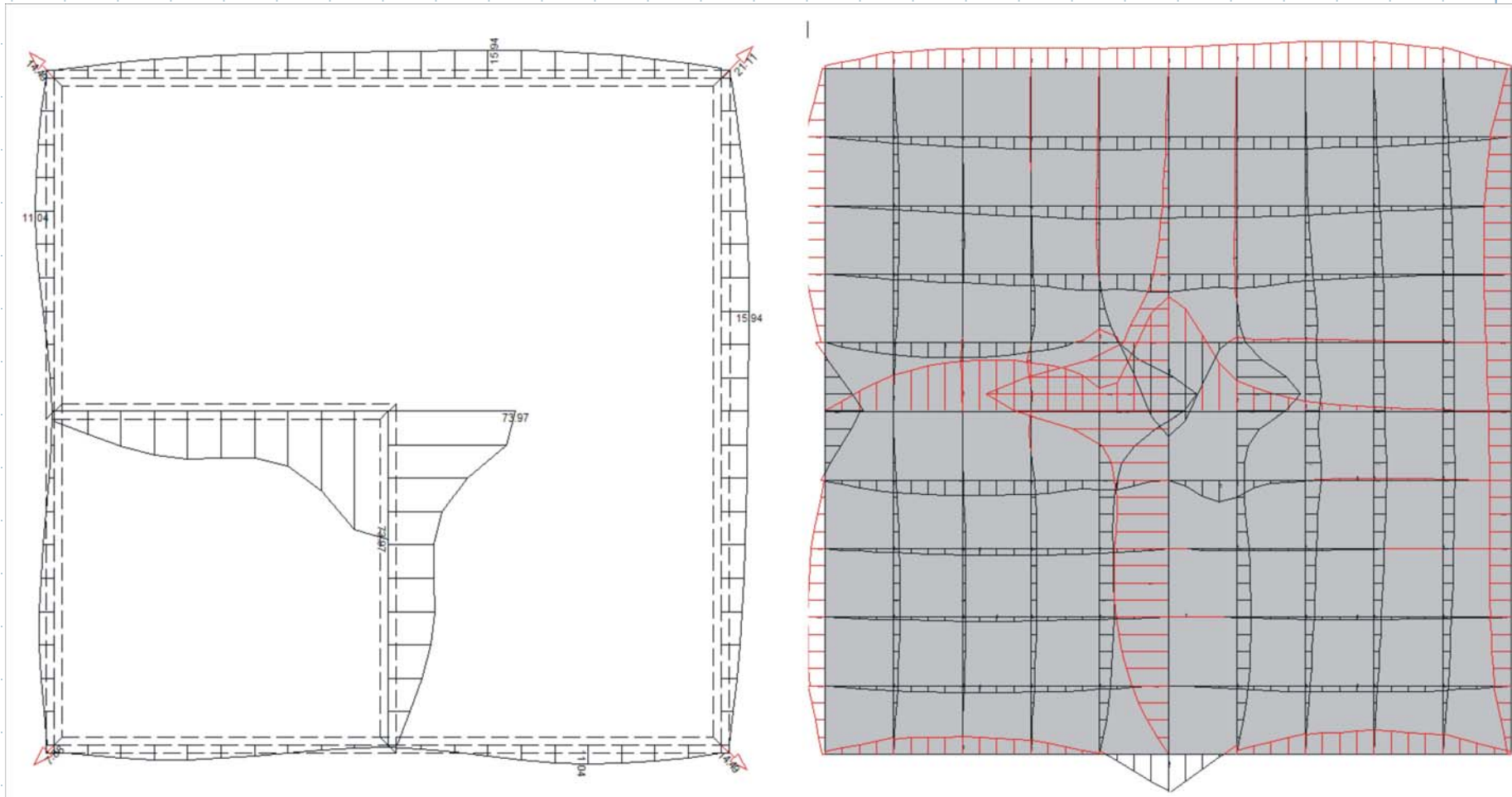
Warum Lagerkräfte oszillieren



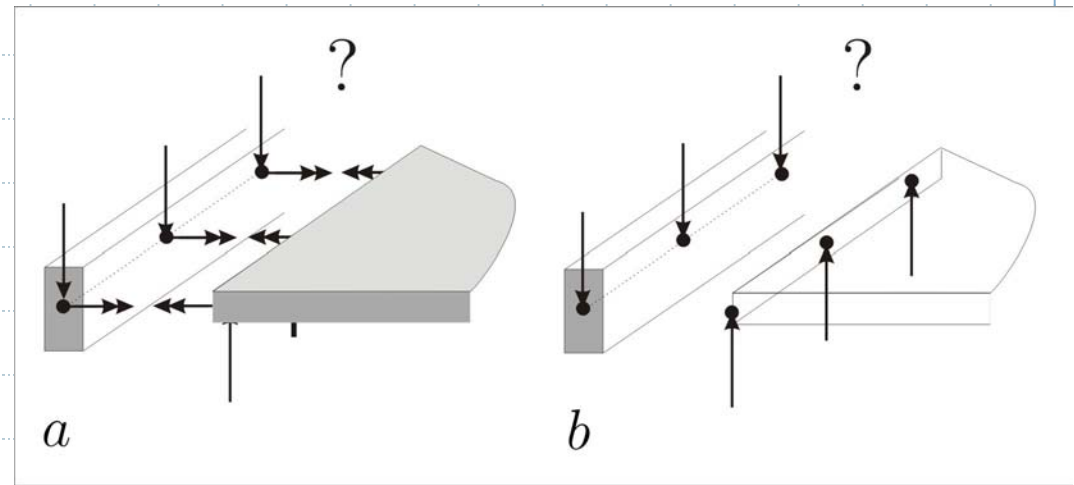
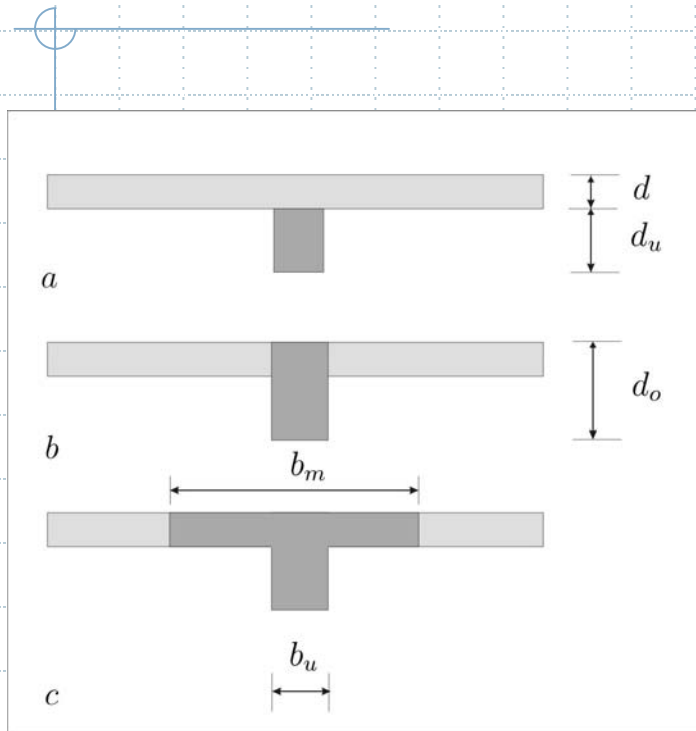
Querkräfte



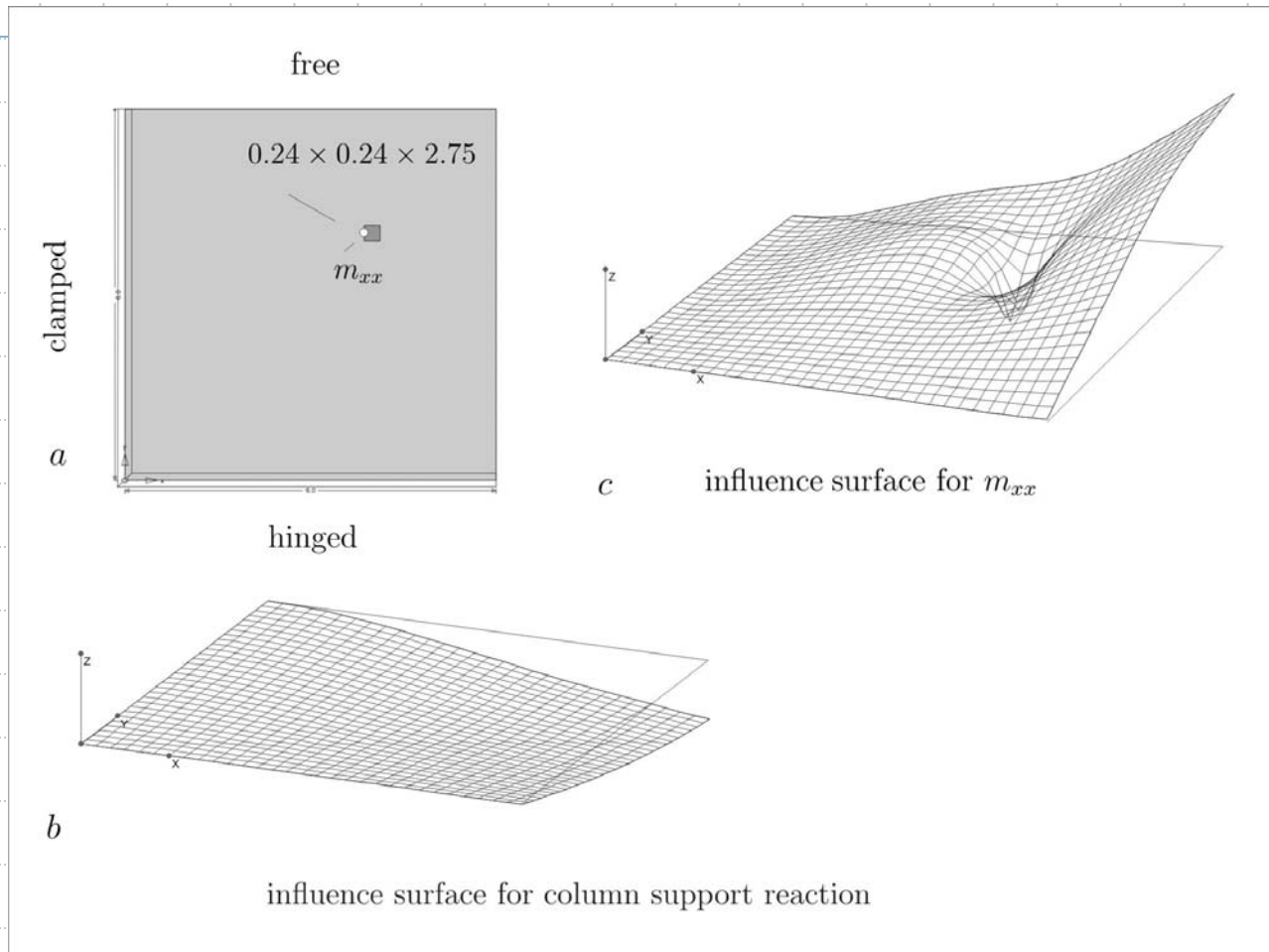
Bildschirm und Realität



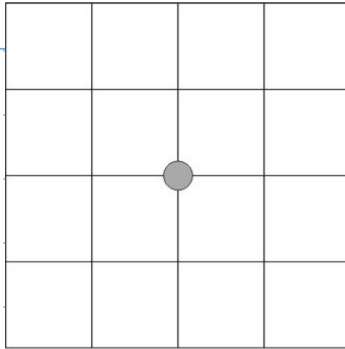
Plattenbalken



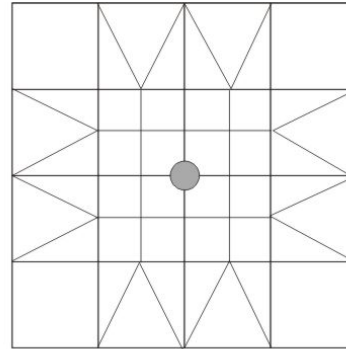
Stützen



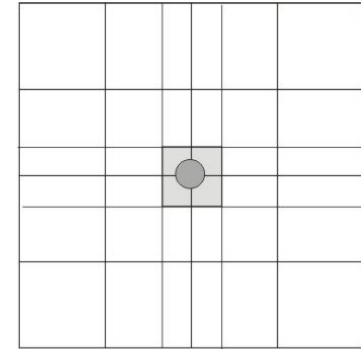
Anpassung des Netzes



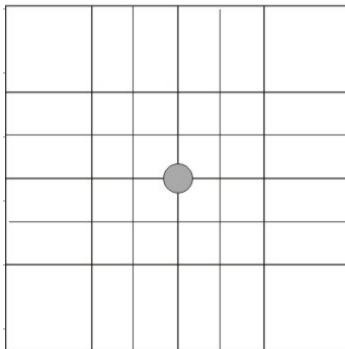
a



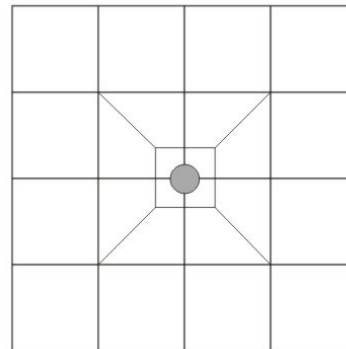
b



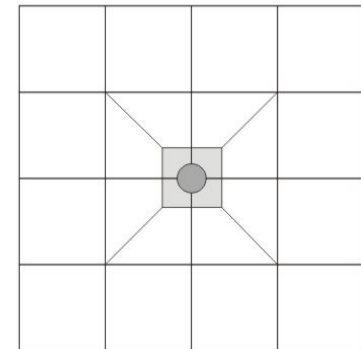
c



d

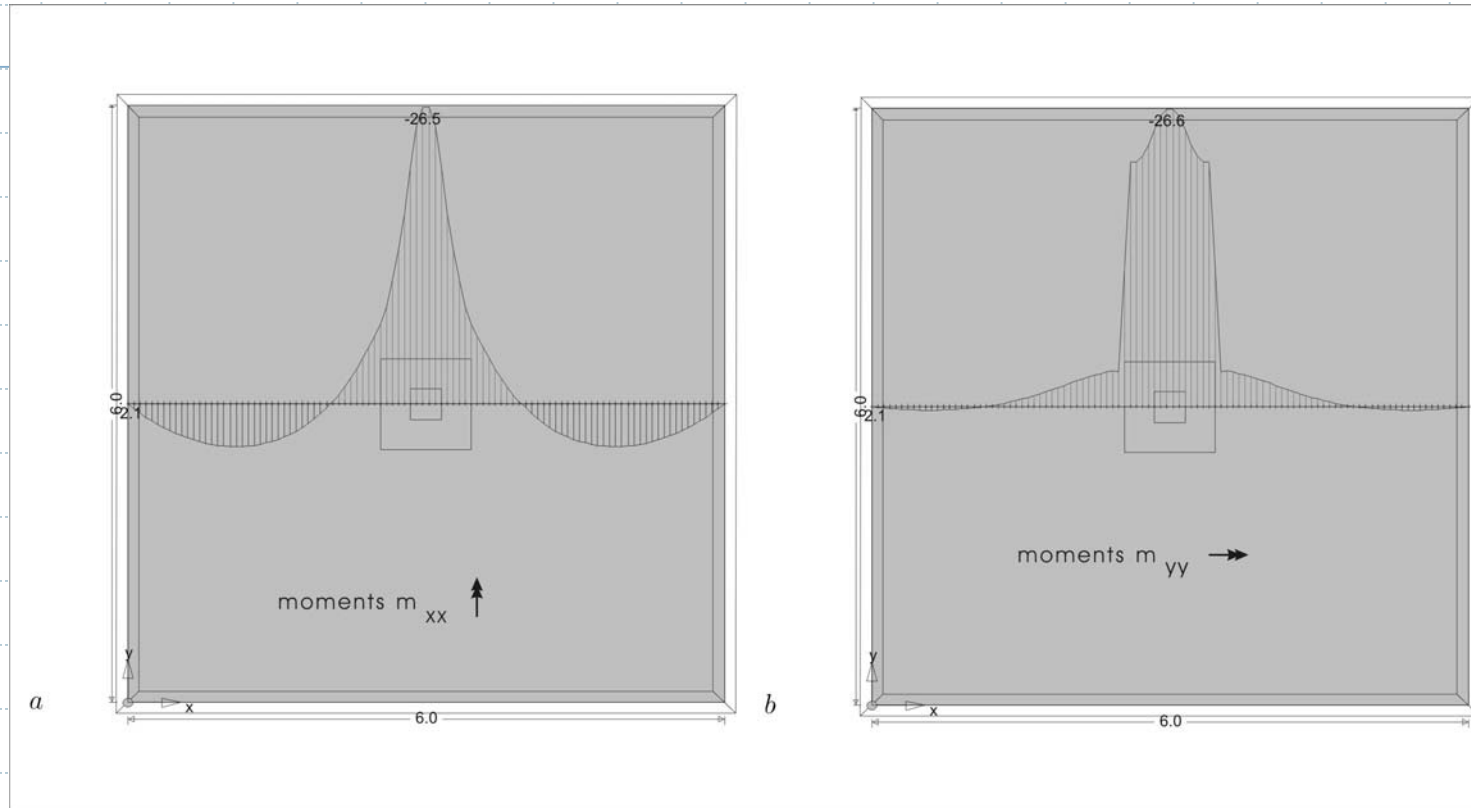


e

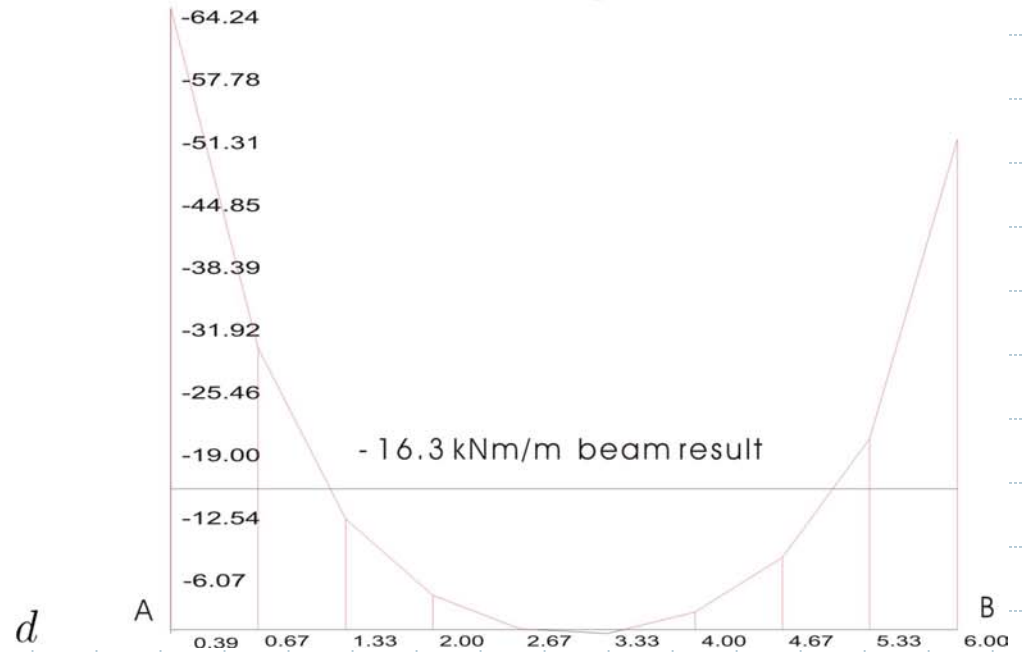
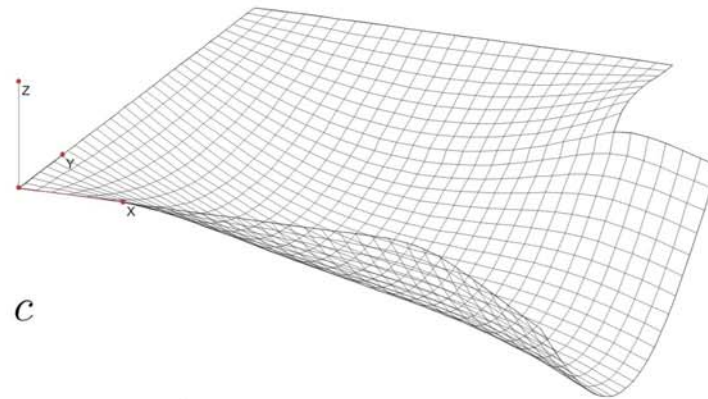
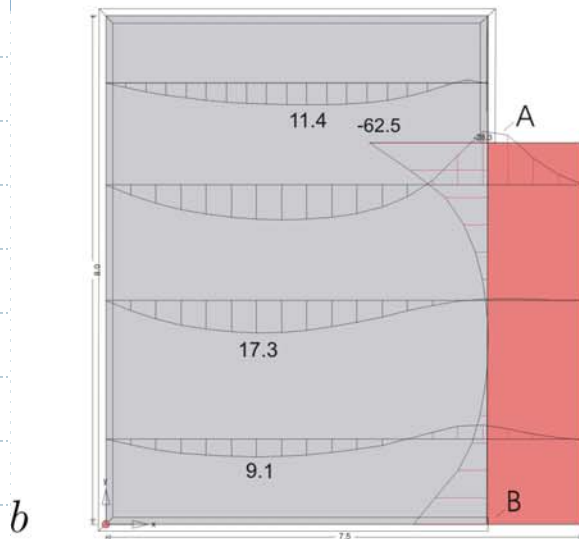
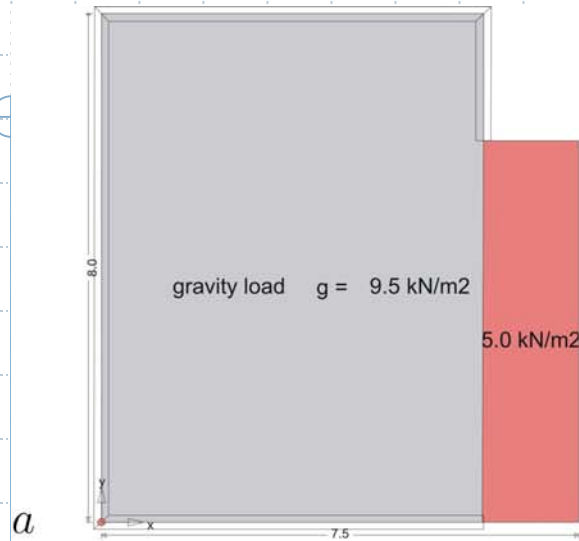


f

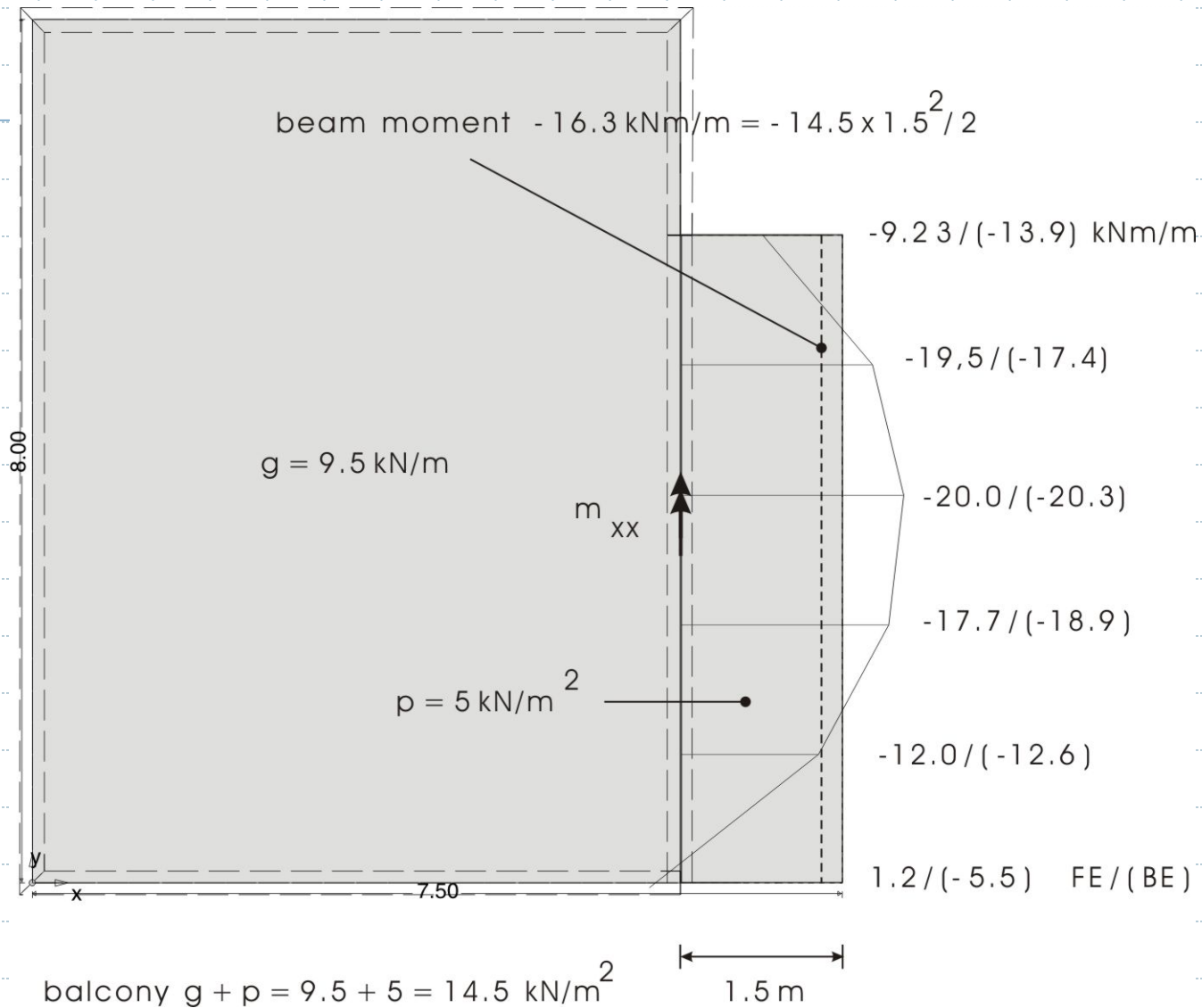
Stützenanschnittsmomente



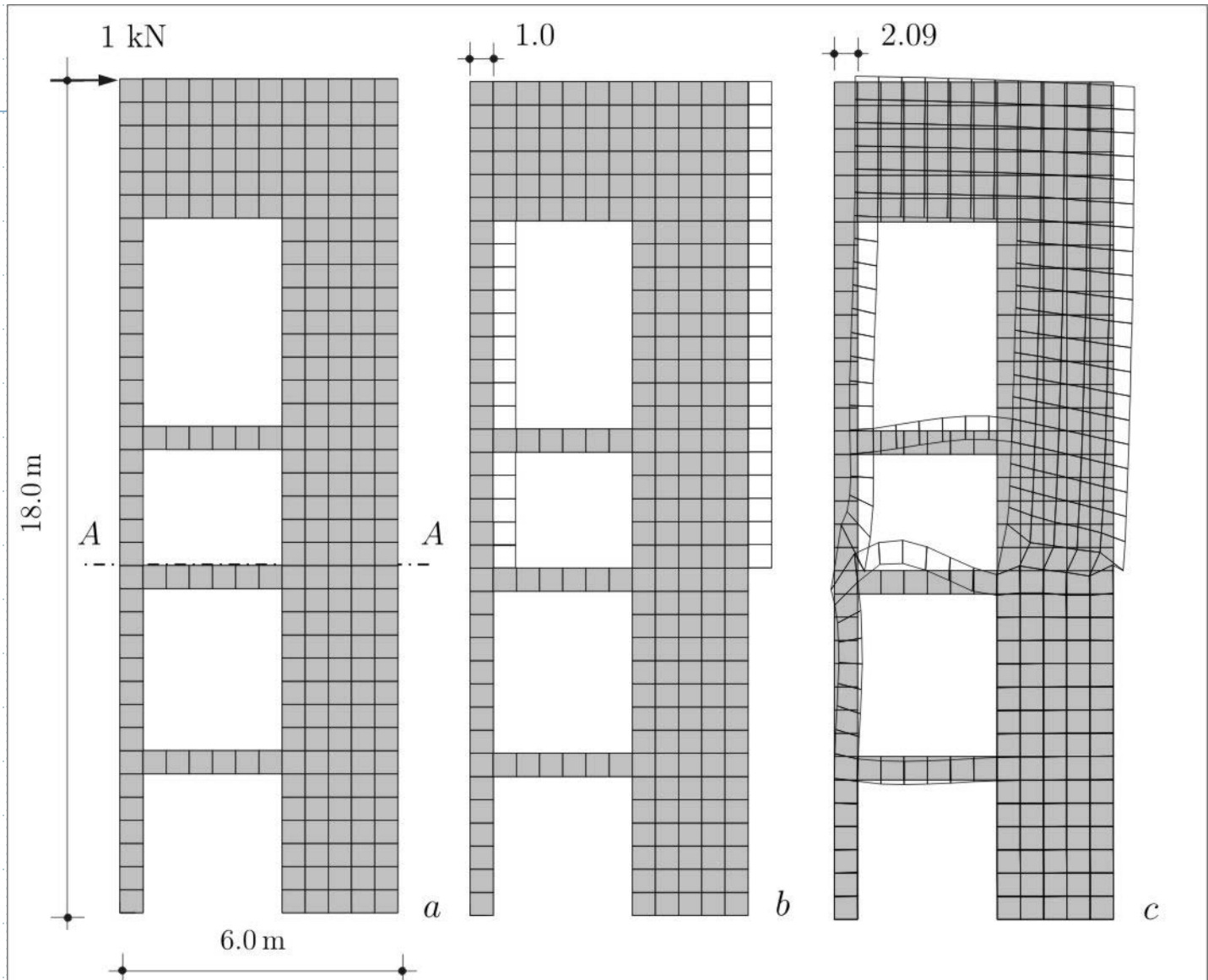
Balkenmodelle - Vorsicht



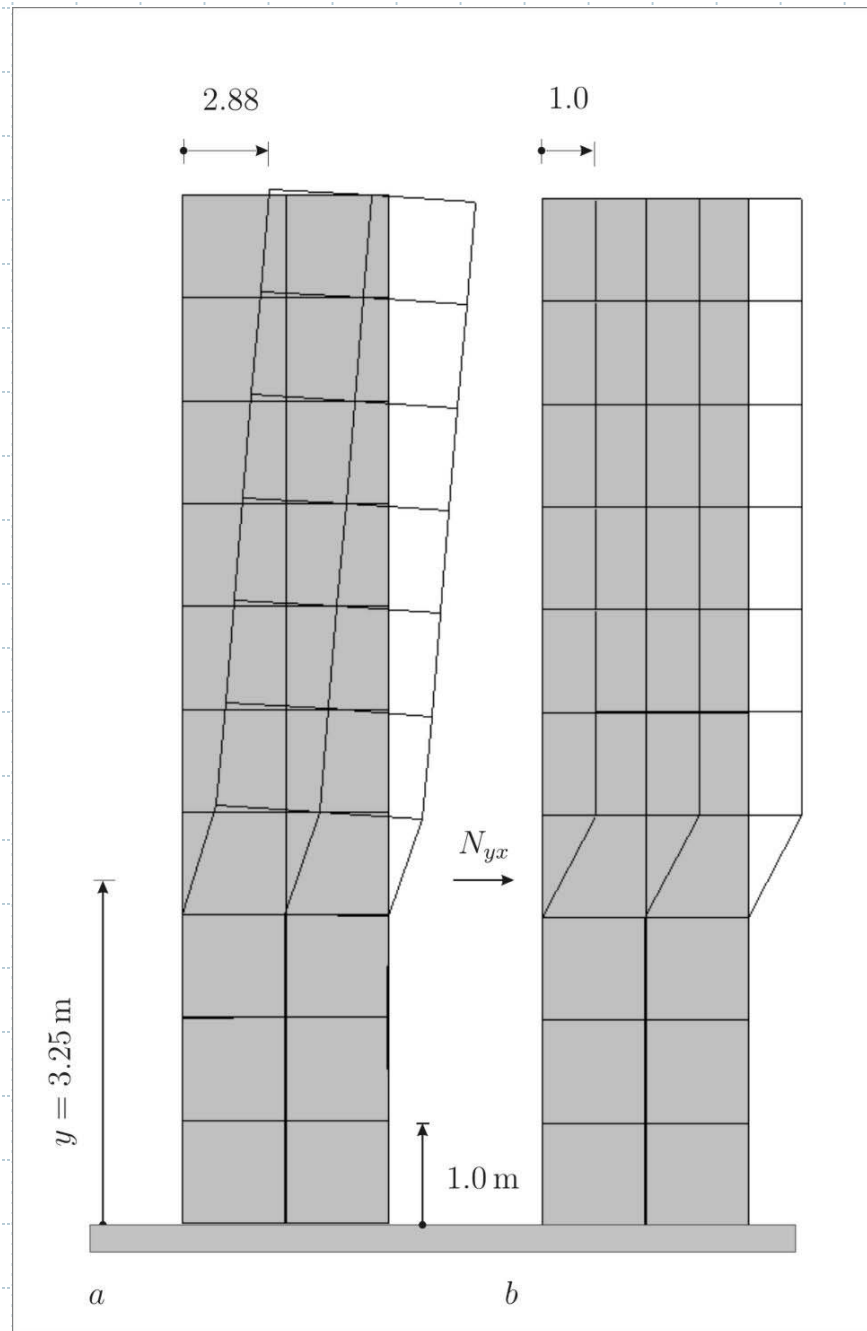
Balkenmodell – Teil 2



Summe $H = 0$



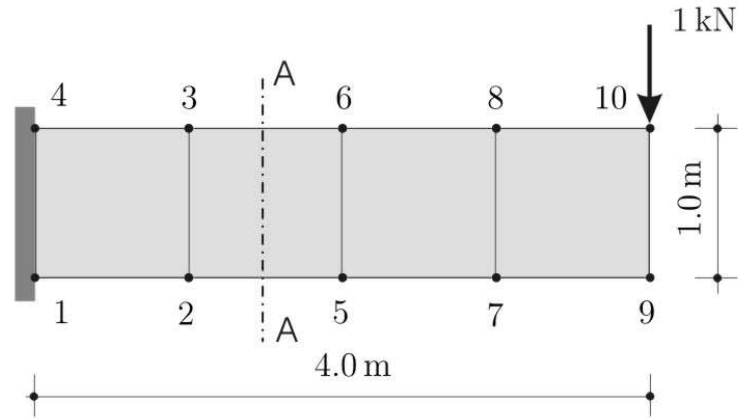
Wilson's Element



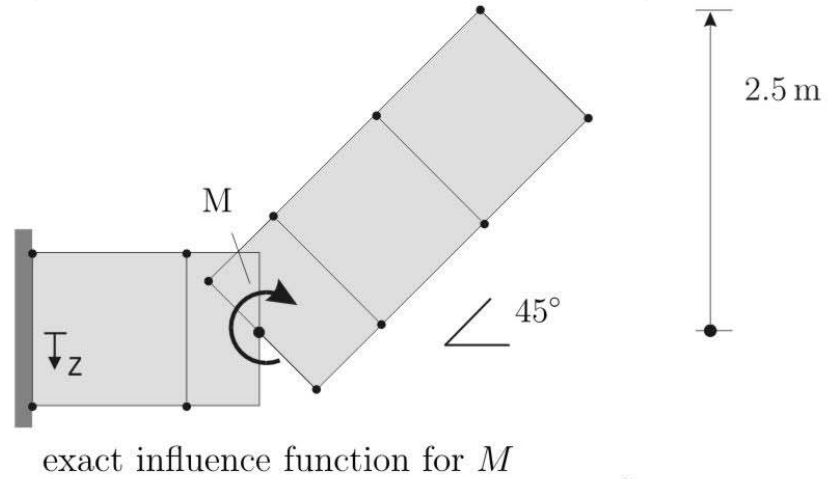
Summe $M = 0$?



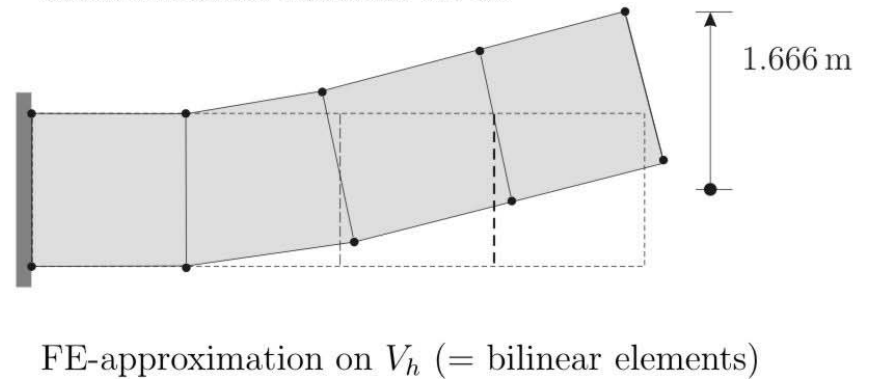
a



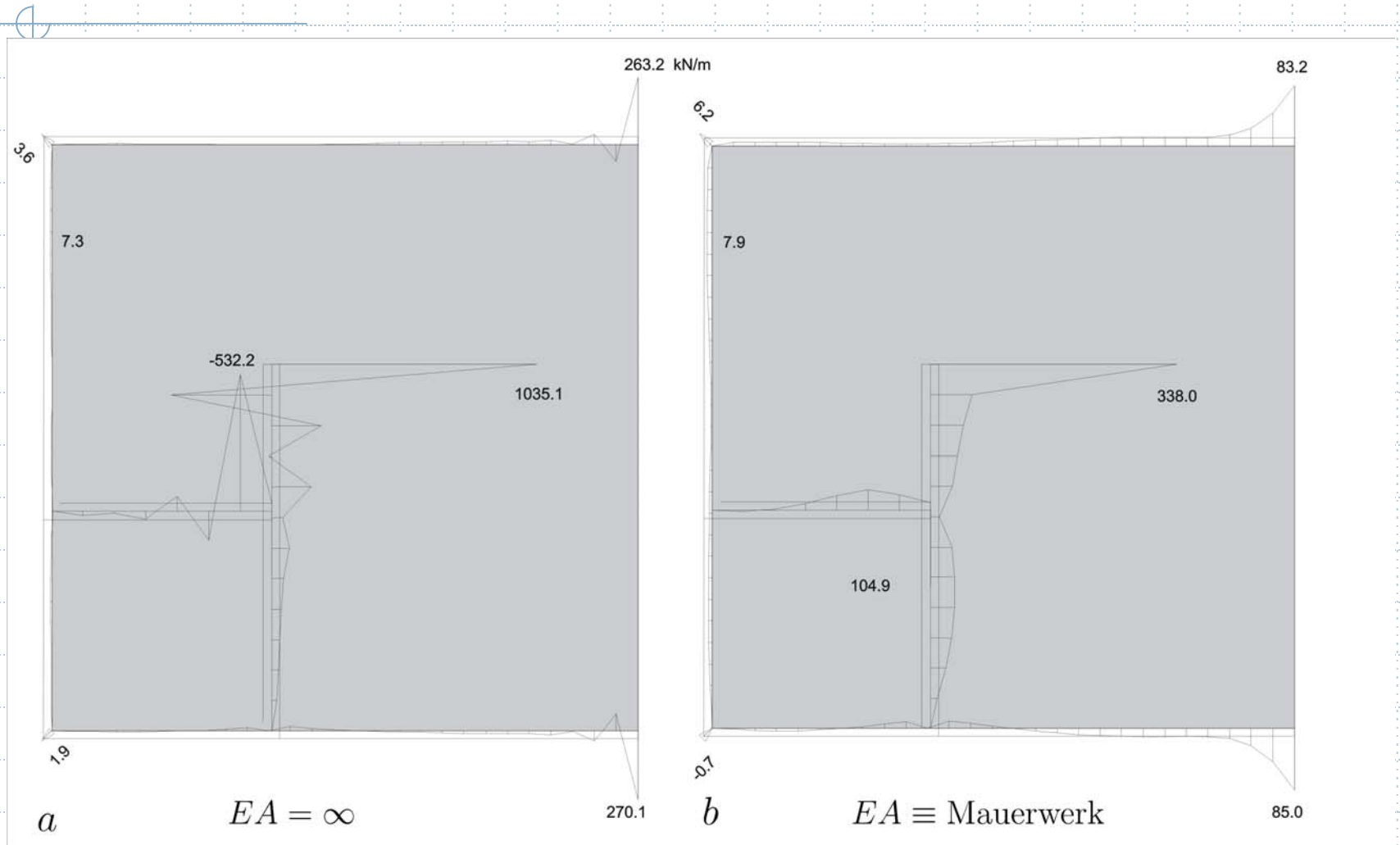
b



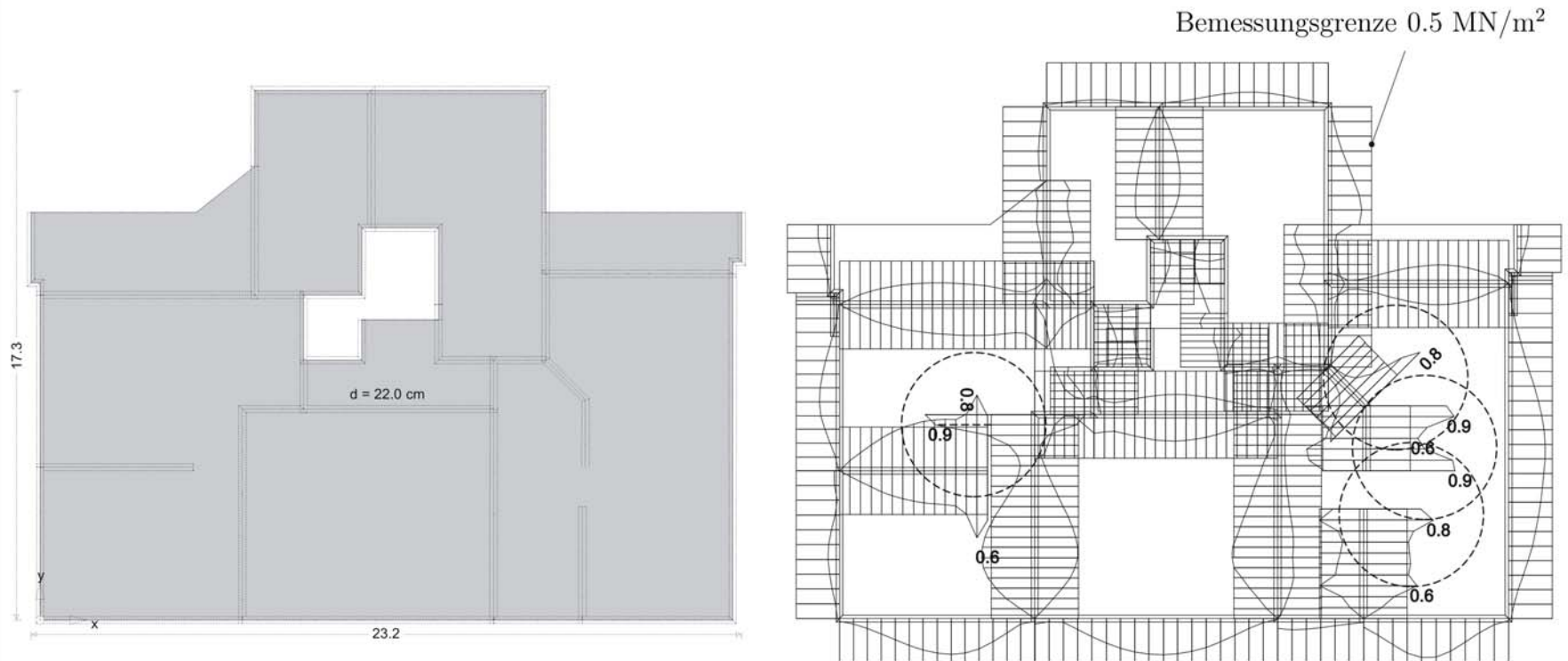
c



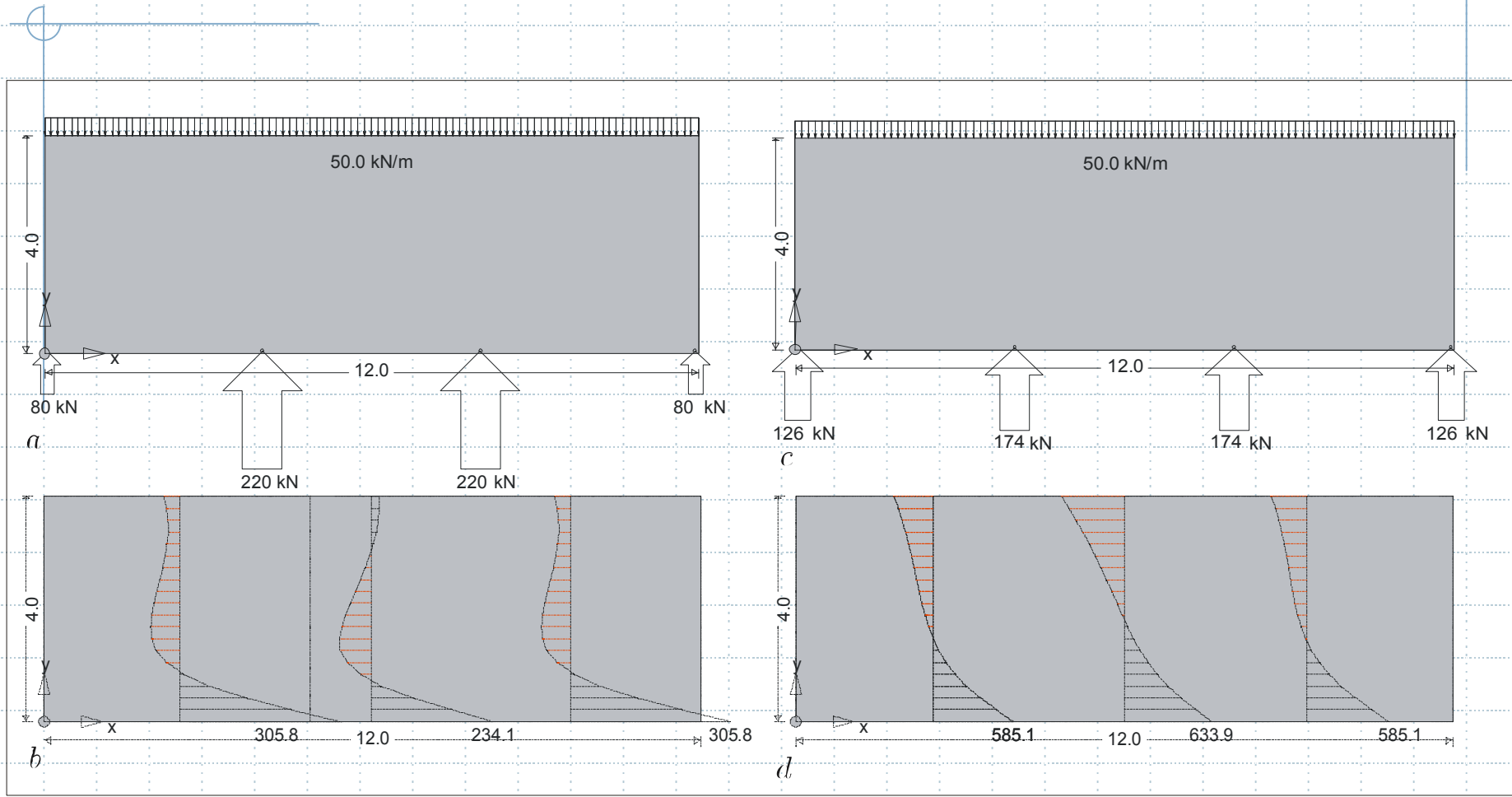
Steifigkeiten



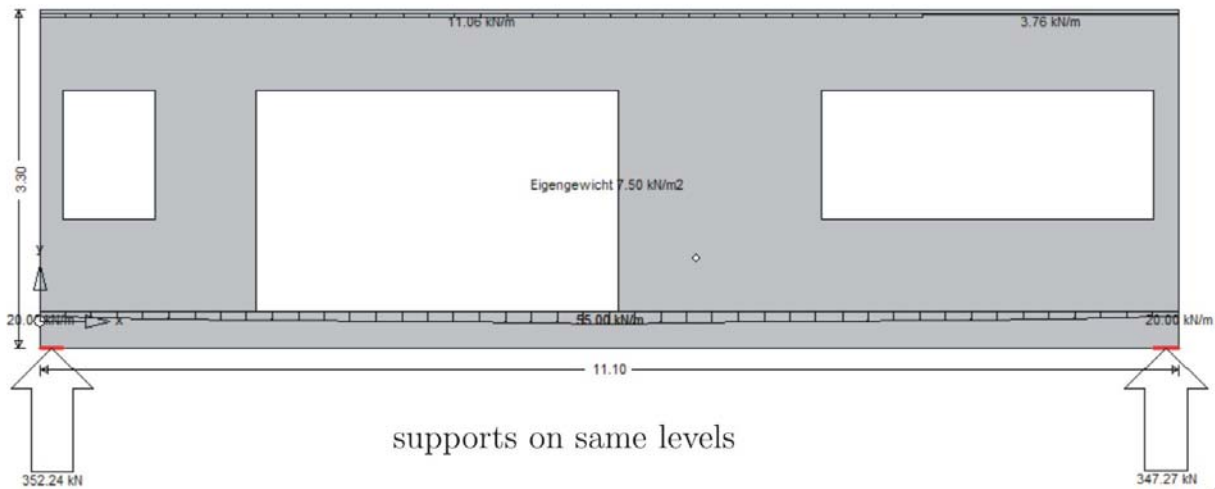
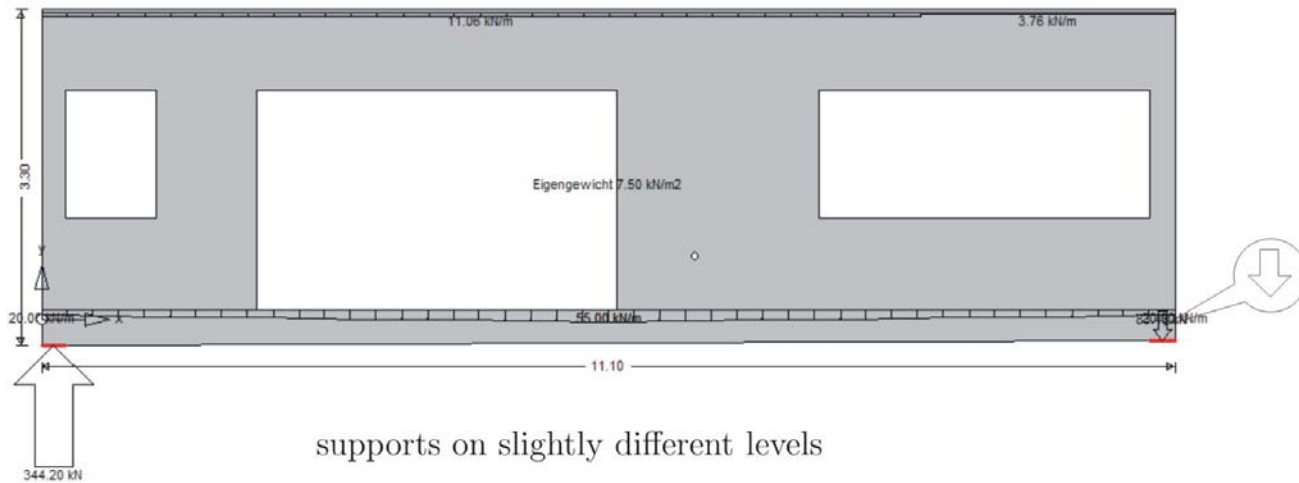
Schub



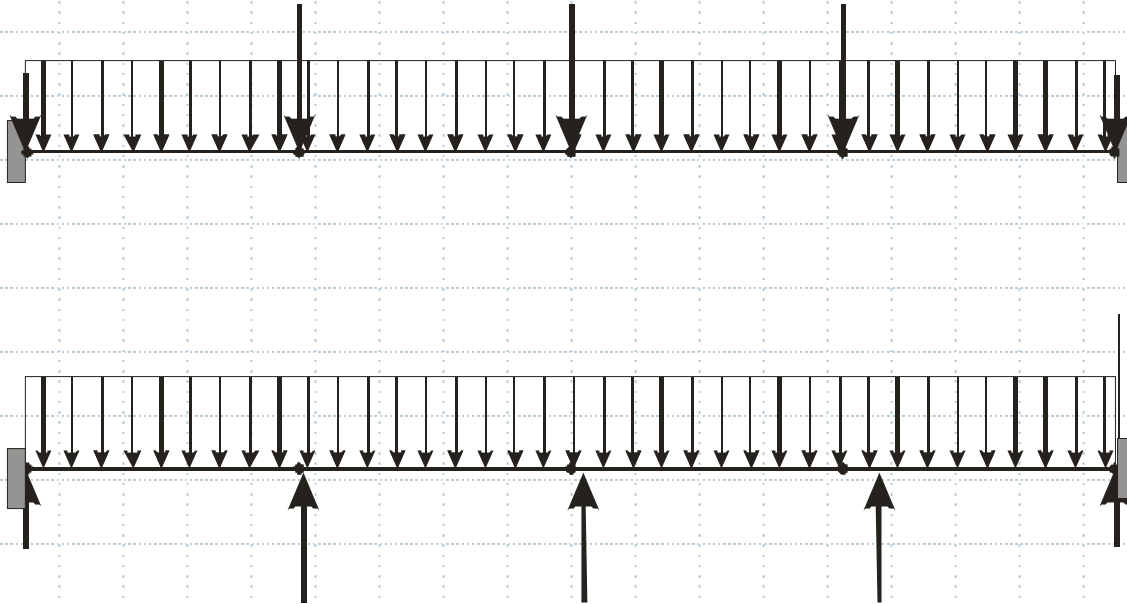
Lagersteifigkeiten bei Scheiben



Versatz von Lagern

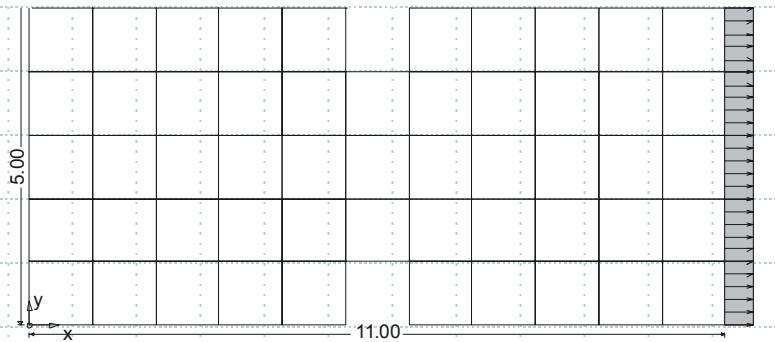


Korrektur $p - p_h$?



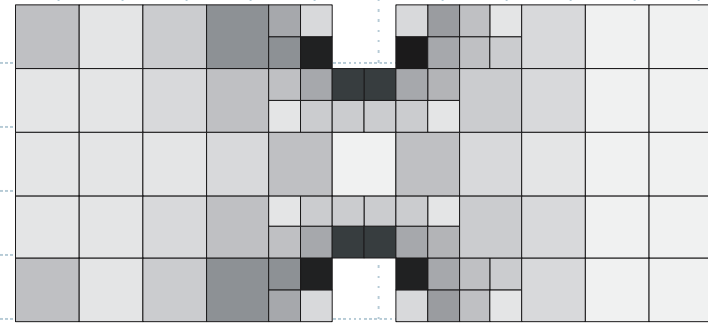
Adaptive Verfeinerung

system and loading

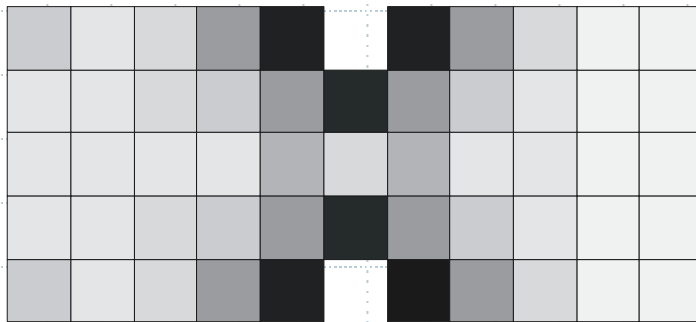


10 kN/m

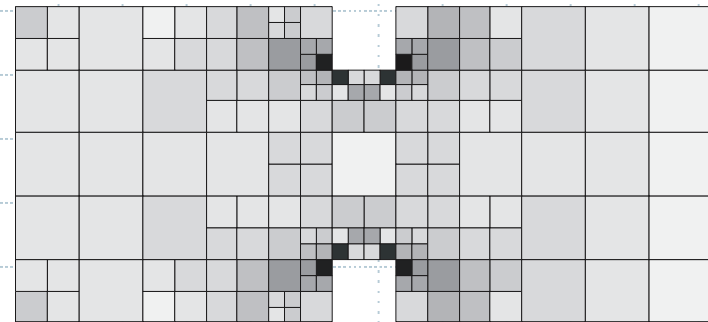
after one refinement



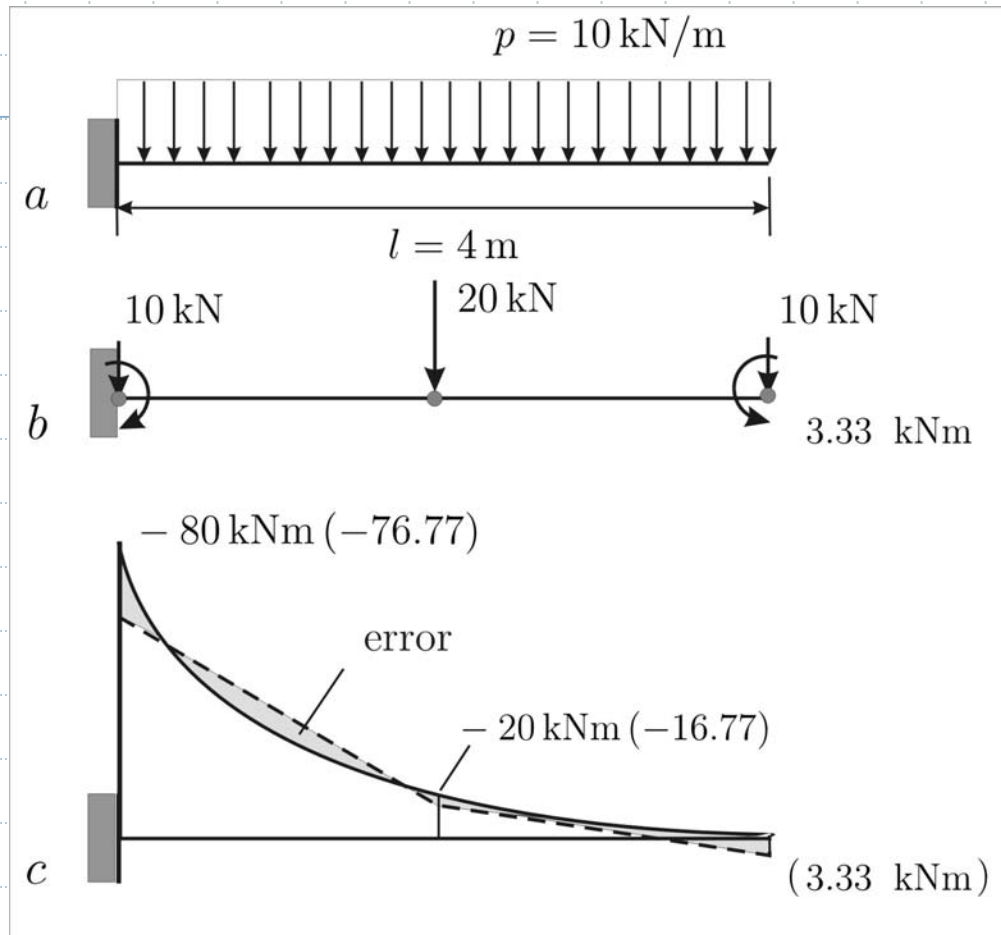
first error plot



after two refinements



Das Kernproblem



Wie kann man aus den Fehlern in den Lasten auf die Fehler in den Schnittkräften schließen?

Die Vergangenheit

- ◆ Freude über die neuen Möglichkeiten

Die Gegenwart – viel Numerik

- ◆ Verification = whether the equations were solved rightly

Die Zukunft – wieder mehr Statik

- ◆ Validation = whether the right equations were solved.

Weitere Informationen

- ◆ www.winfem.de
- ◆ Hartmann/Katz Statik mit finiten Elementen, Springer-Verlag
- ◆ Grätsch, T. , Hartmann, F.: Über ein Fehlerbild bei der Schnittgrößenermittlung mit finiten Elementen, Teil 1: Scheiben. Bautechnik 78 (2001), S. 327-332.
- ◆ Grätsch, T. , Hartmann, F.: Über ein Fehlerbild bei der Schnittgrößenermittlung mit finiten Elementen, Teil 2: Platten. Bautechnik 79 (2003)
- ◆ Grätsch, T., Hartmann, F.: Warum ein FE-Programm falsch rechnet, Bauingenieur (2003)