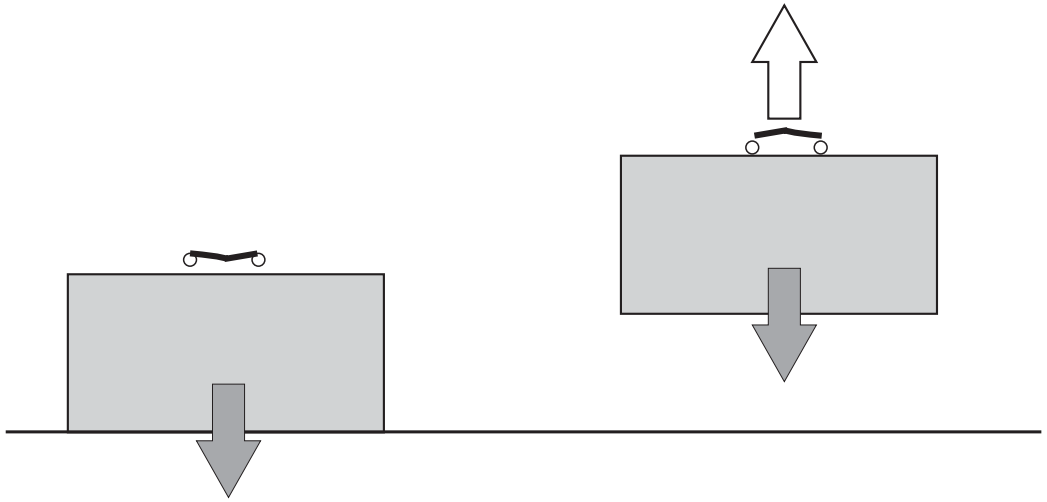
Genauigkeit und Fehler bei finiten Elementen

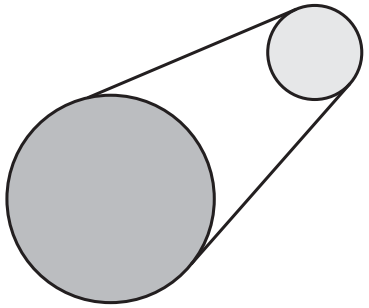
F. Hartmann

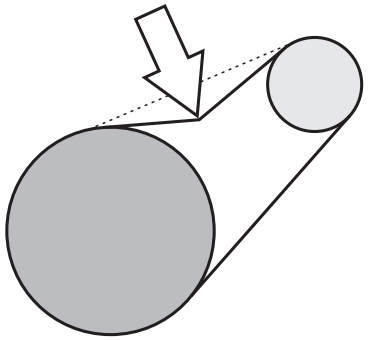
Institut für Baustatik und Baudynamik
Universität Kassel

21. Juni 2009

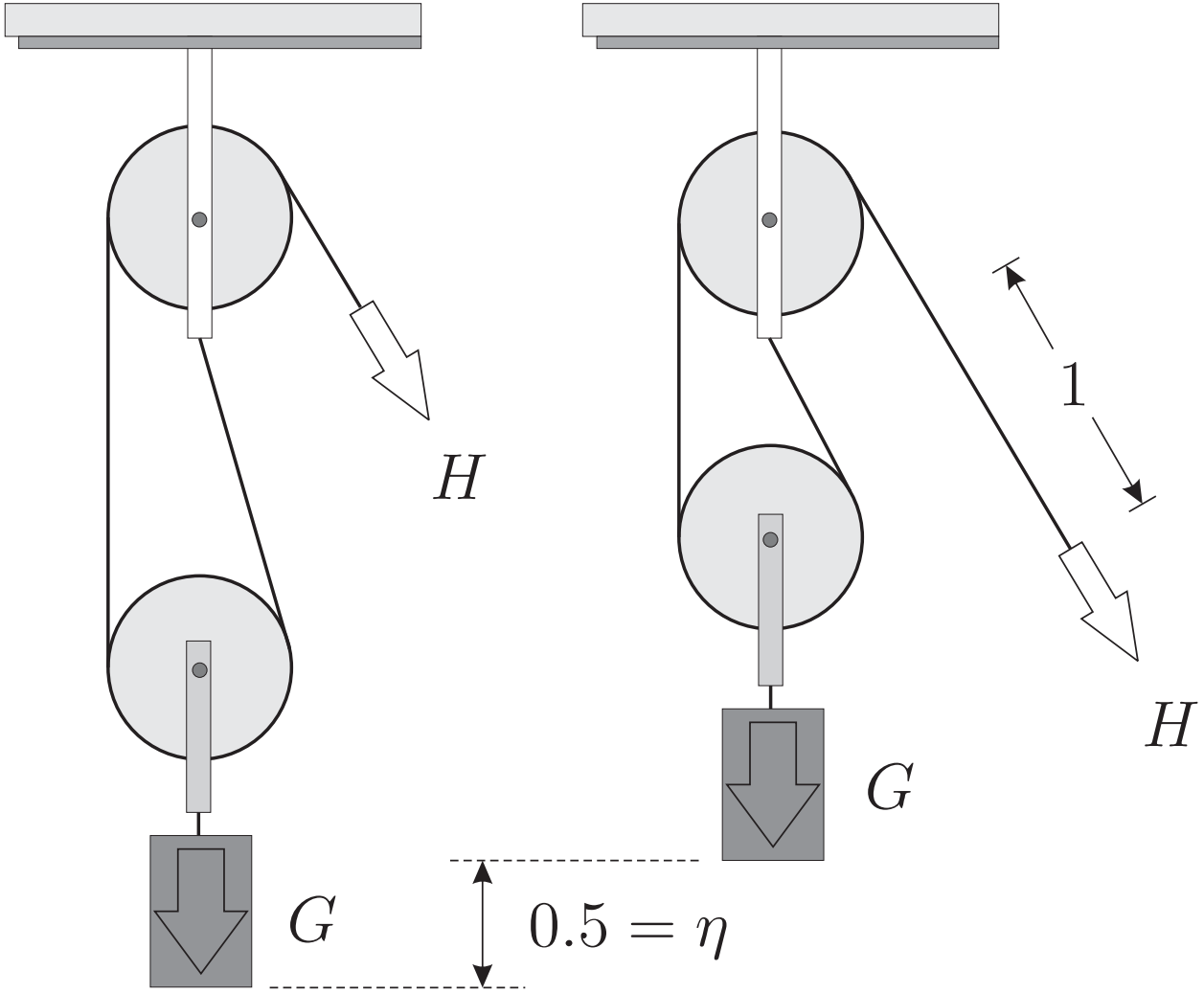
Messen = Bewegen



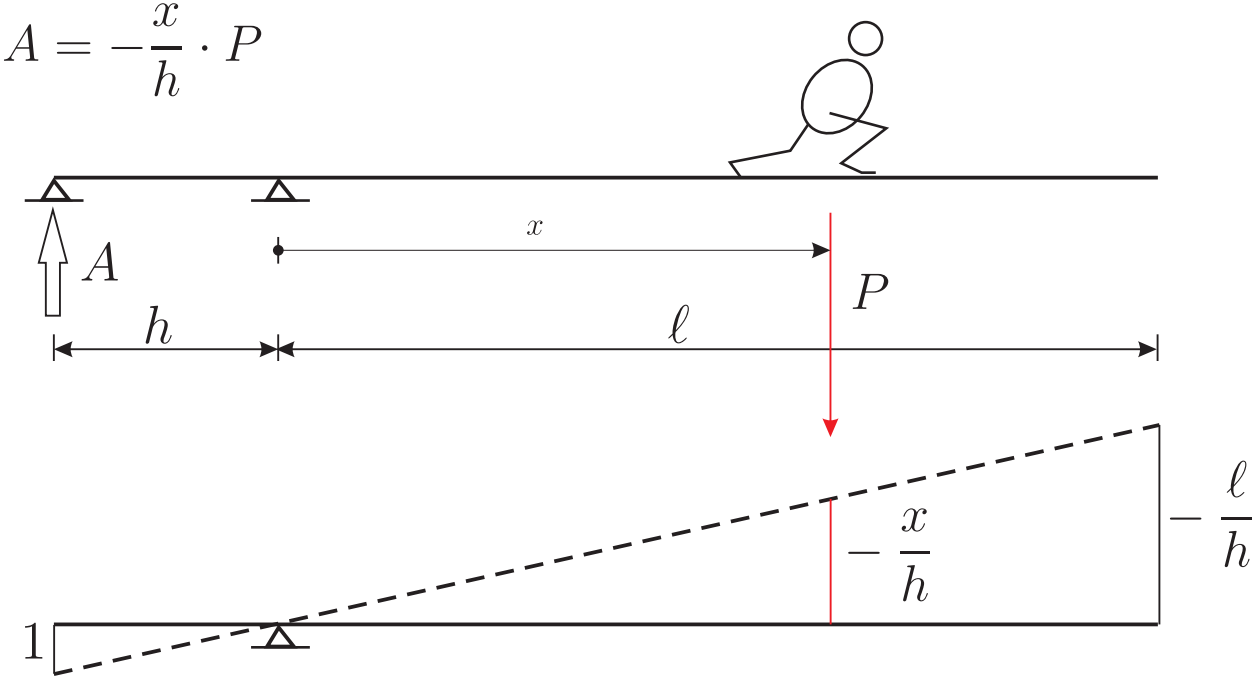


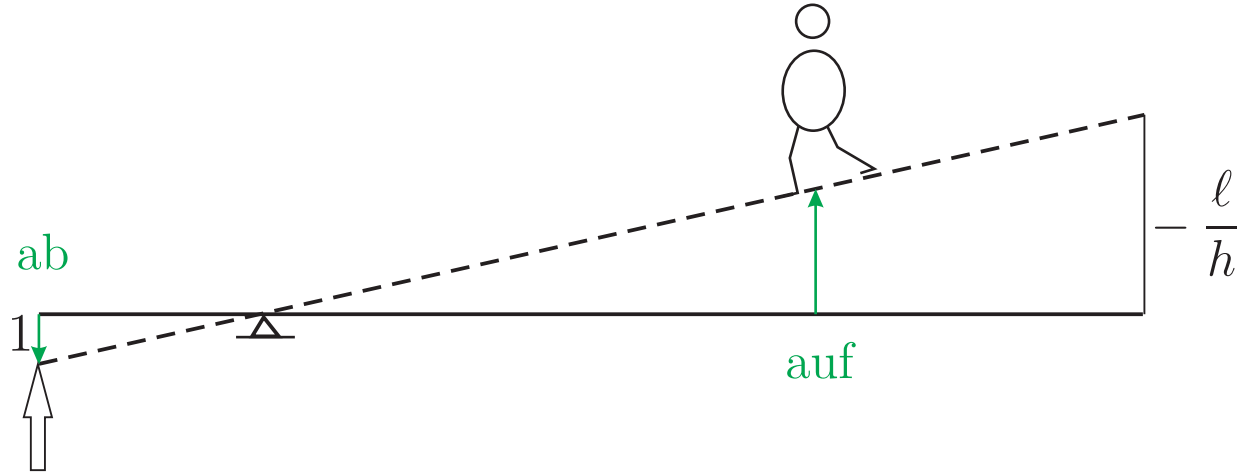


Was kommt bei der Last an?

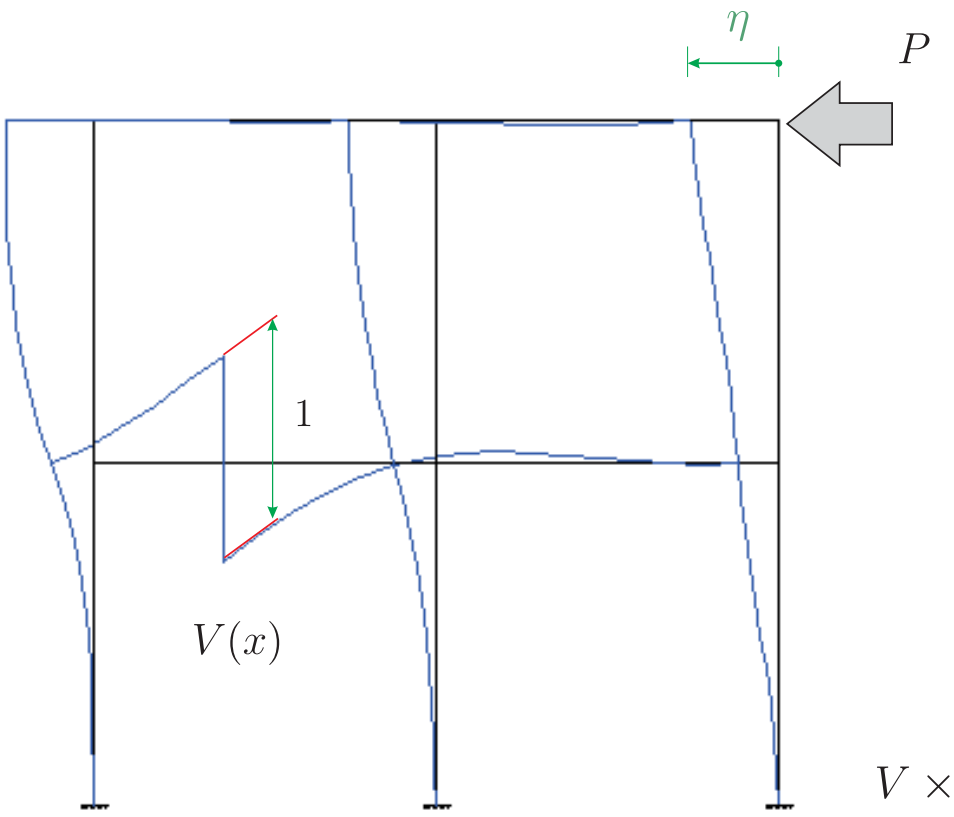


$$A = -\frac{x}{h} \cdot P$$

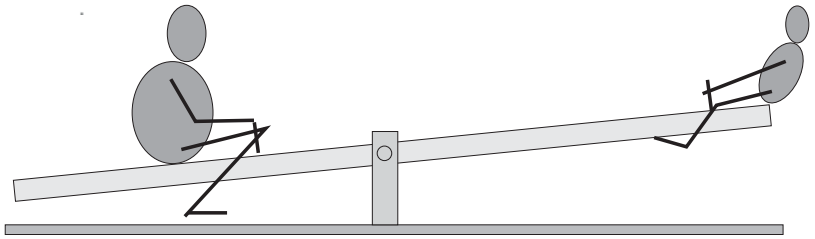




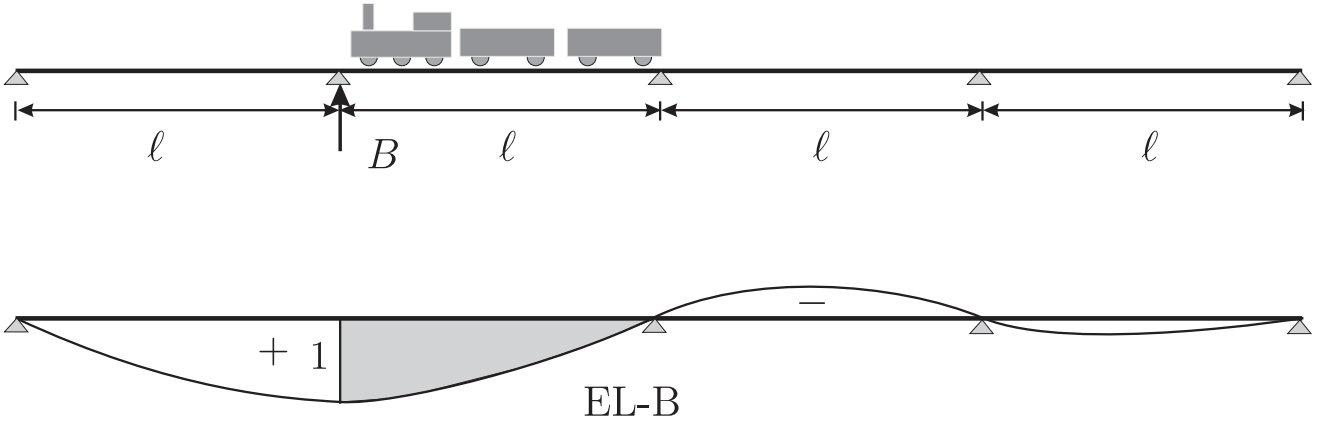
Zu jeder Schnittgröße gehört eine Schaukel.



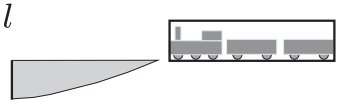
$$V \times 1 = P \times \eta$$



Eisenbahn auf Brücke

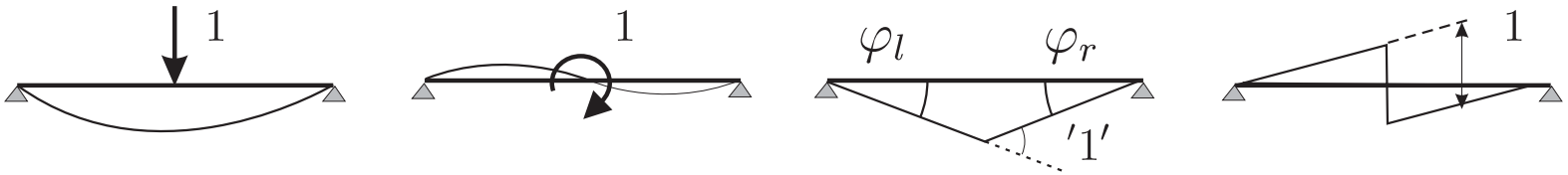


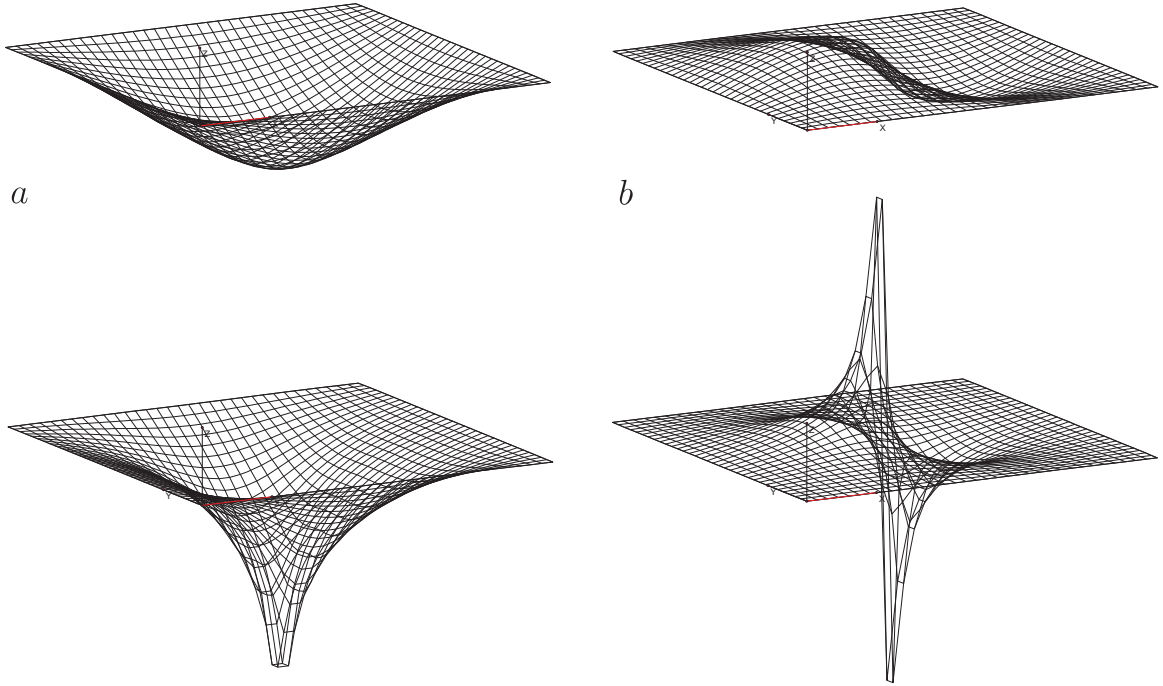
$$B \times 1 = \int_0^l G_0(y,x)\, p(y)\, dy = \int_0^l G_0 \cdot \text{Belastung} = \int_0^l$$



$$dy$$

$$\text{output} = \int_0^l \text{Einflussfunktion} \cdot \text{input}$$


1
1
φ_l
φ_r
'1'
1


a
b

Thesen

Mechanik ist Kinematik.

Zu jeder Zahl gehört eine Bewegung.

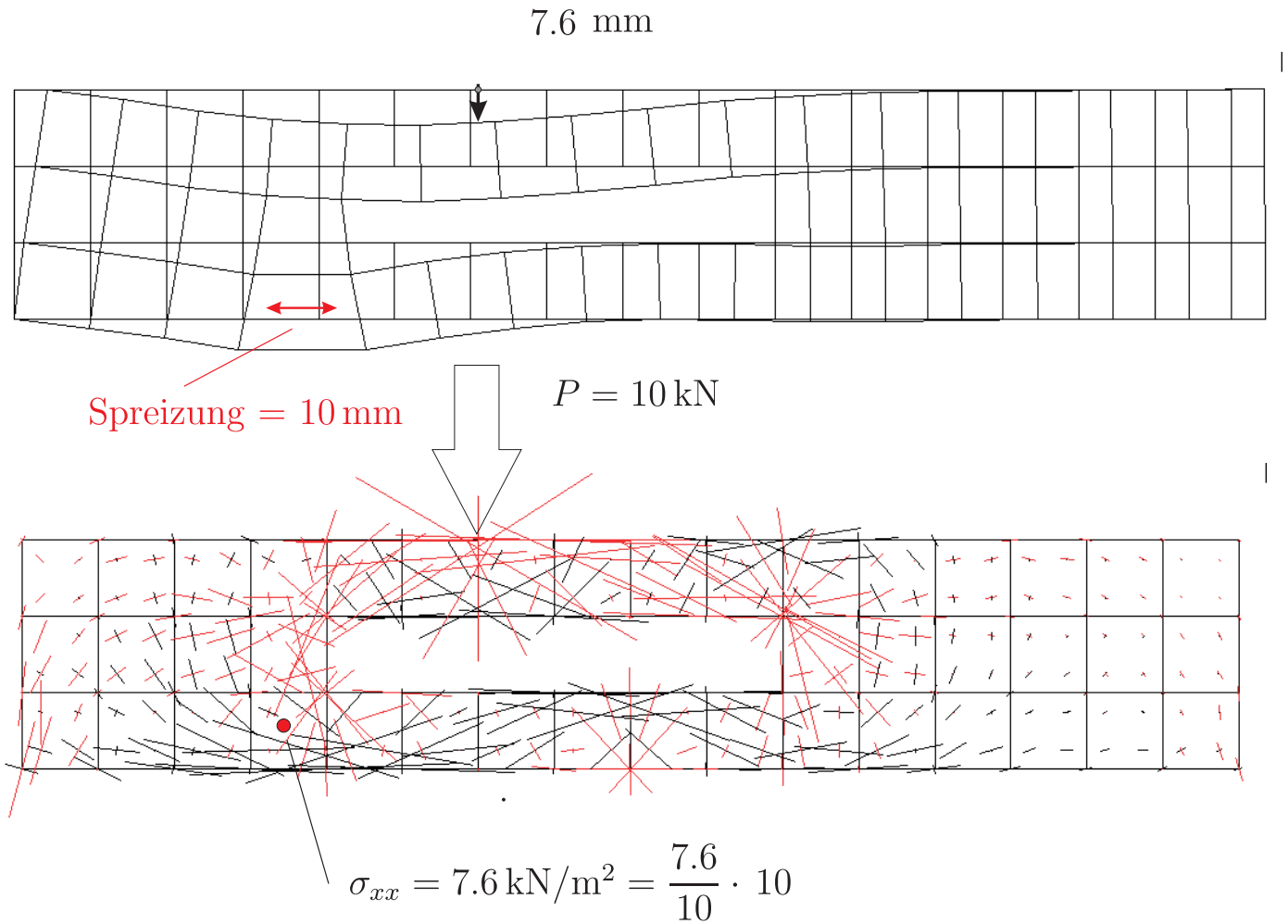
Jede Zahl ist eine Energie.

$$u(x) \cdot 1 = \int_\Omega G(y,x)\, p(y)\, dy \qquad \text{(Betti)}$$

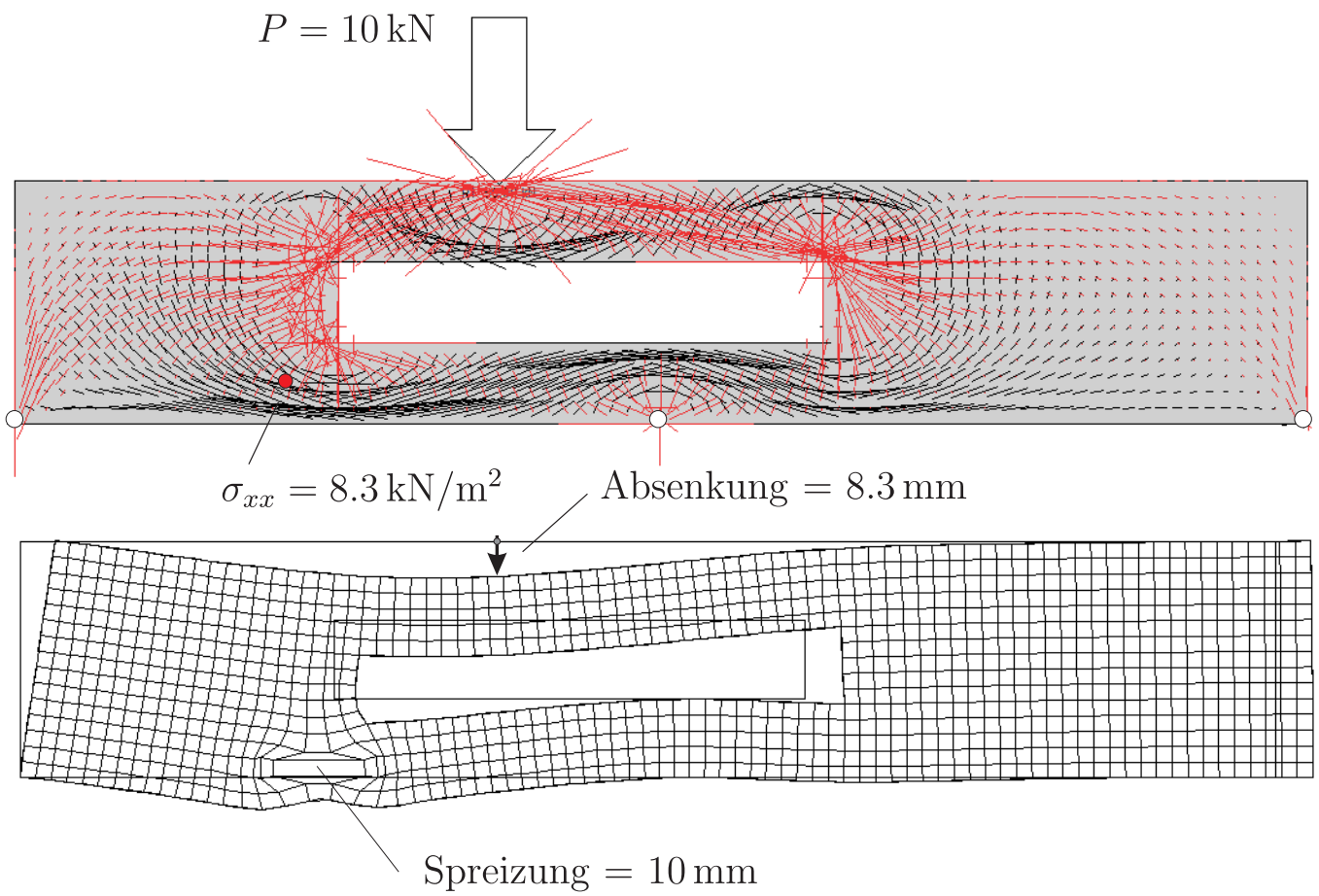
Der zentrale Satz

Ein FE-Programm berechnet alle Werte mittels genäherten Einflussfunktionen.

Spannung in einem Punkt



Verbessertes Netz

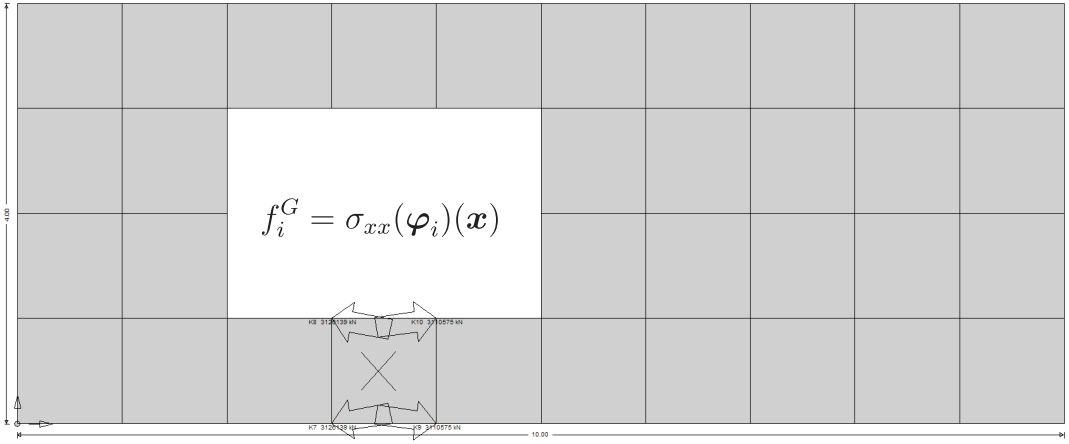


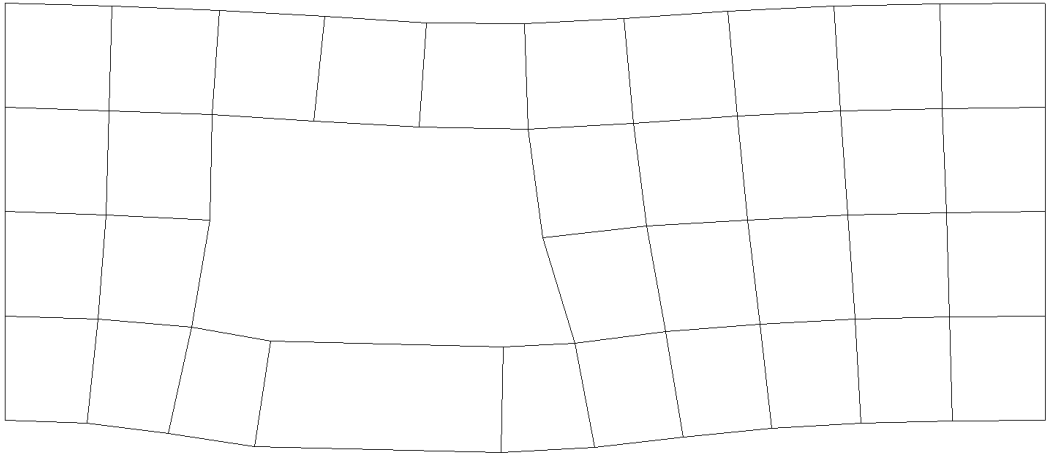
Konsequenz

Mit falschen Einflussfunktionen erhält man falsche Werte.

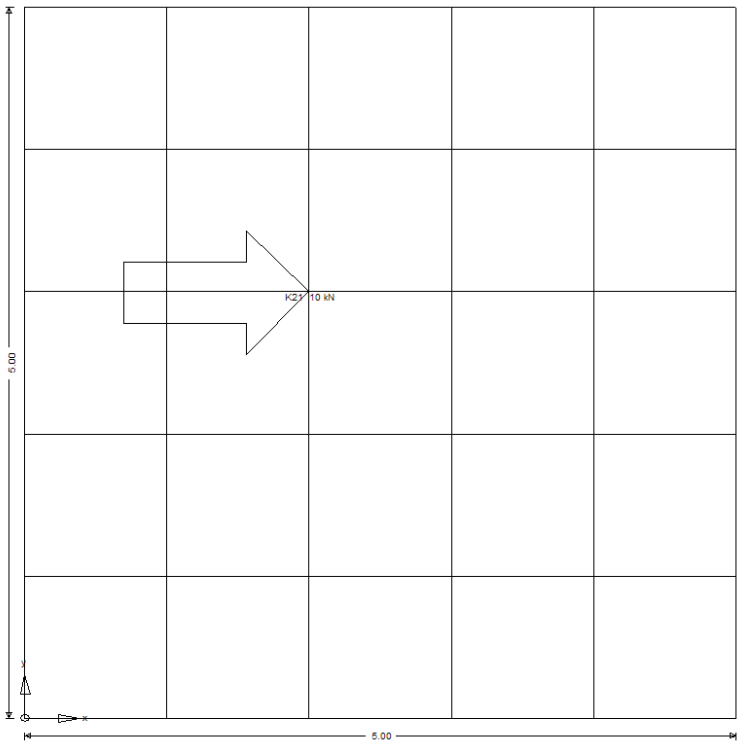
Also müssen die Einflussfunktionen verbessert werden!

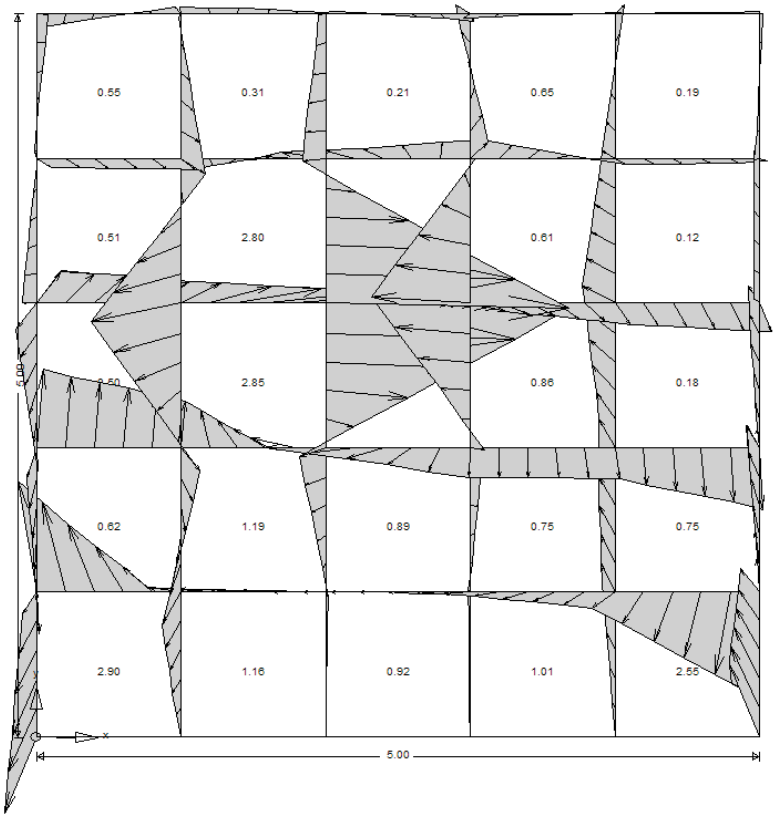
Einflussfunktionen sind normale Lastfälle.



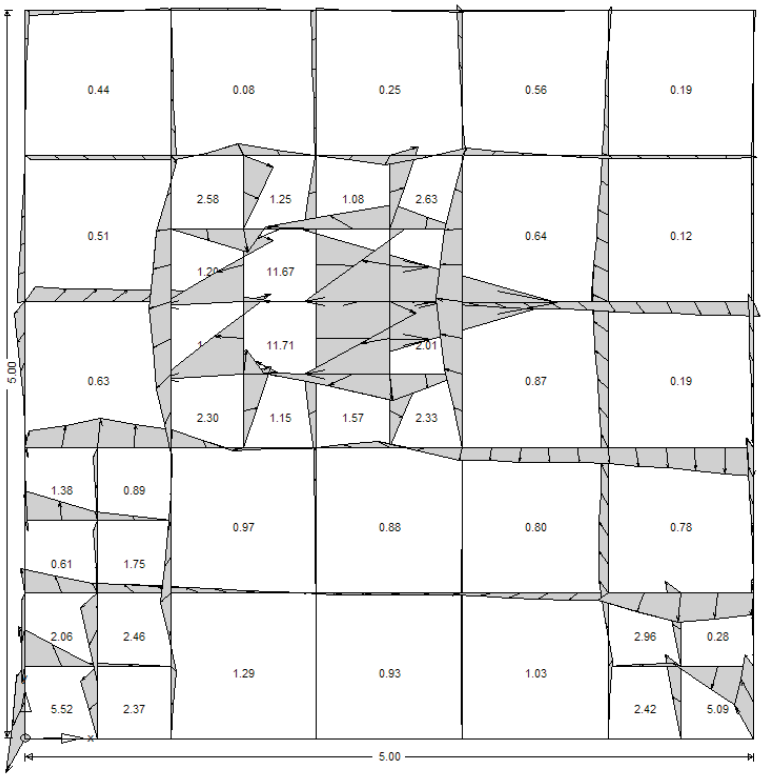


Wie kann man Einflussfunktionen verbessern?

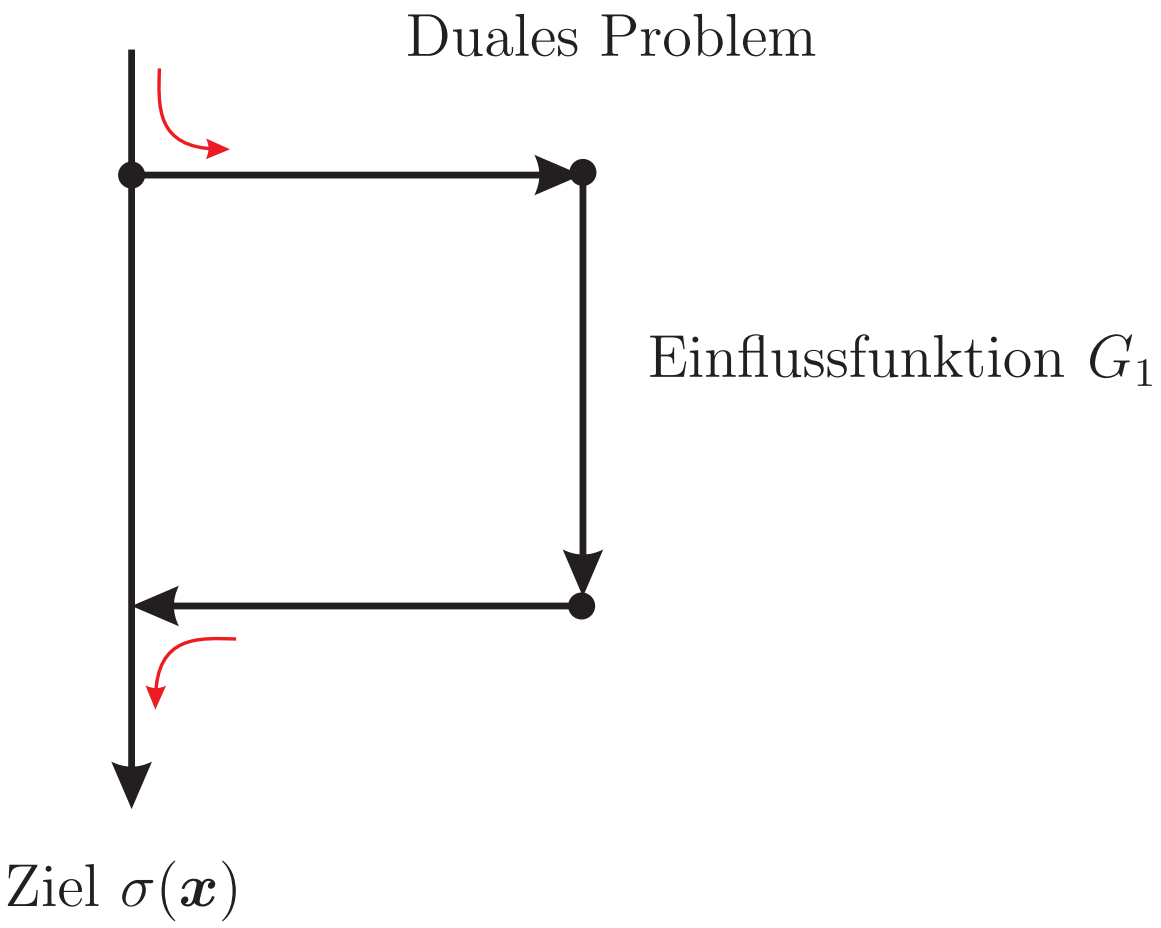




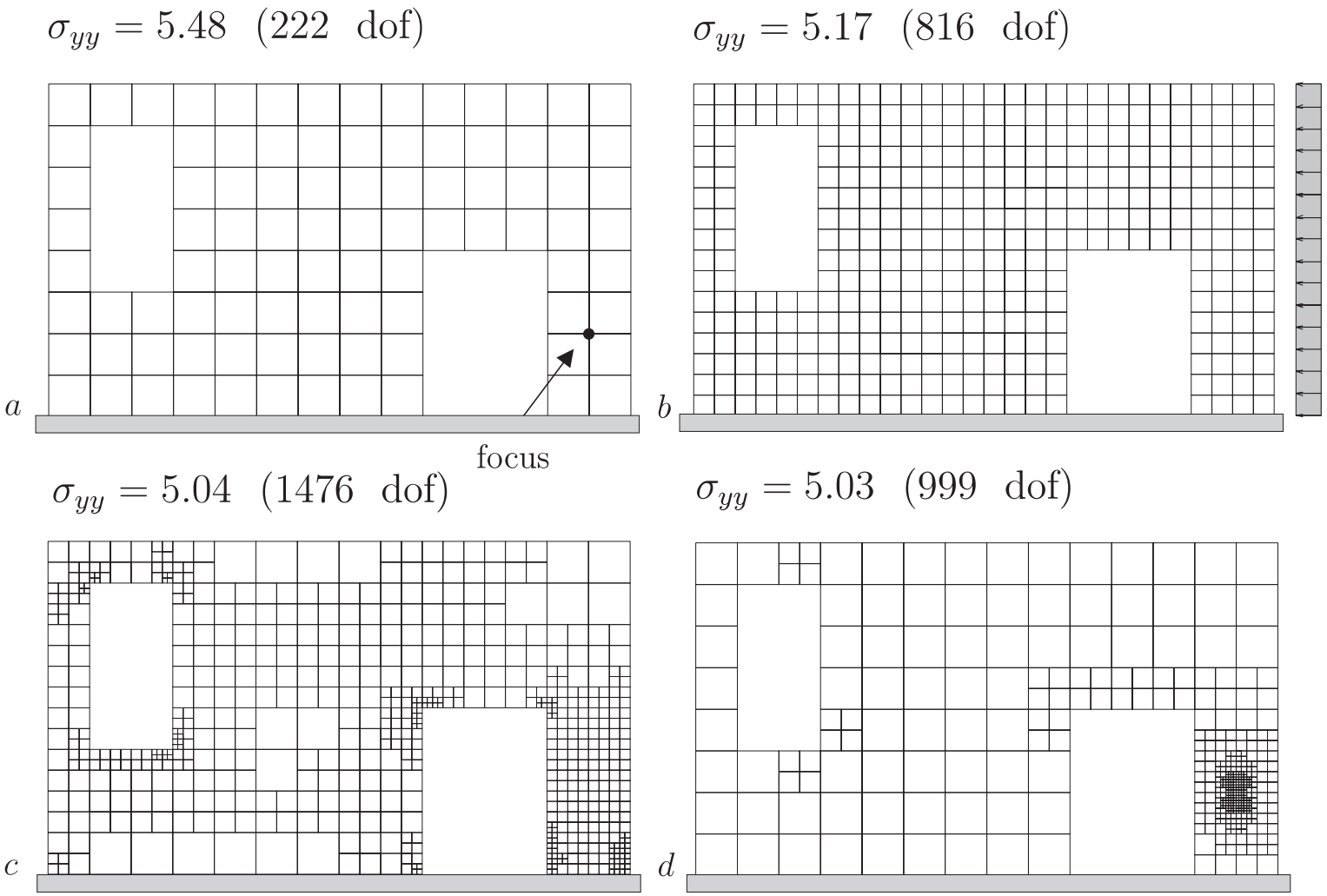
Adaptive Verfeinerung des Netzes



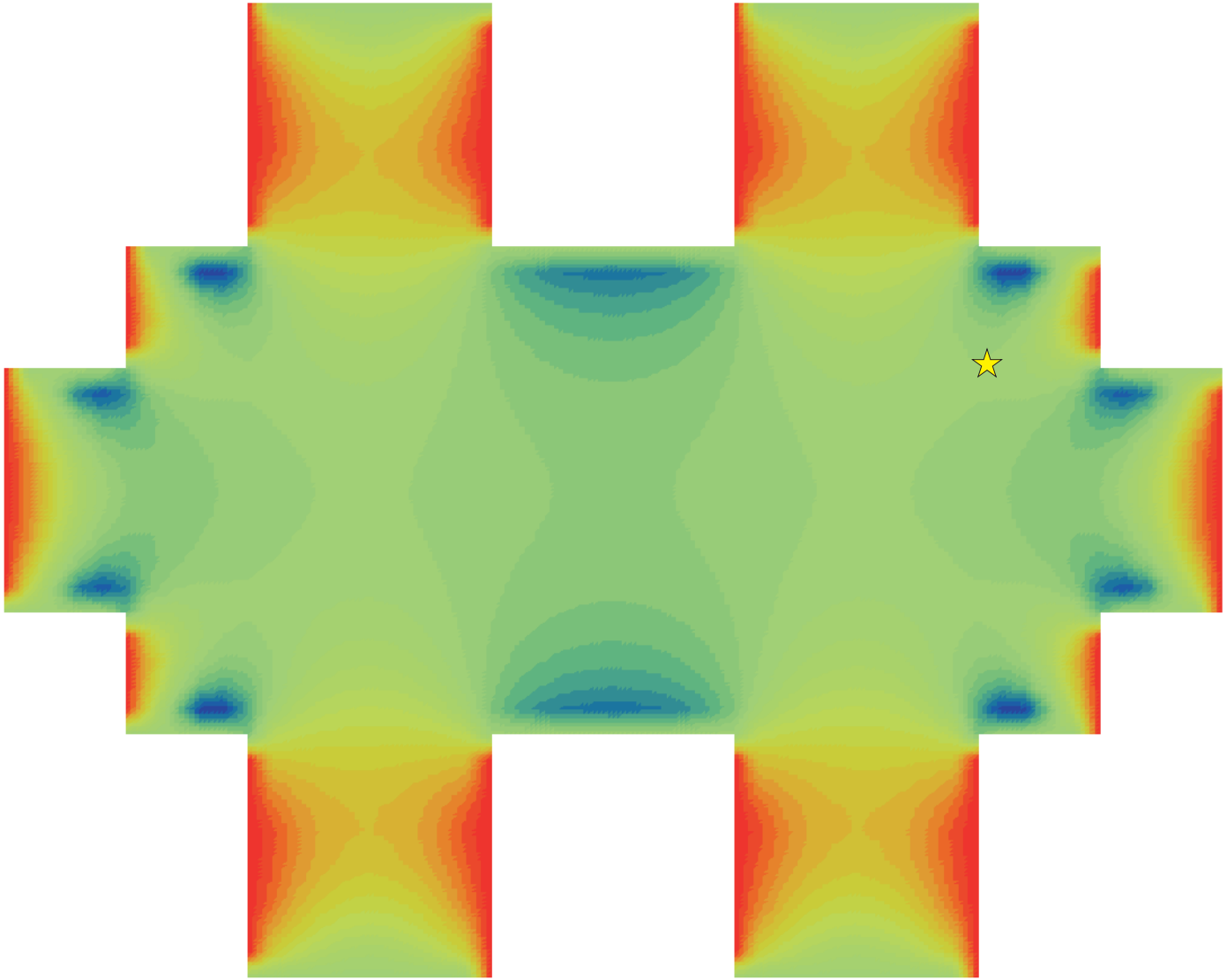
Strategie


Primäres Problem
Duales Problem
Einflussfunktion G_1
Ziel $\sigma(\boldsymbol{x})$

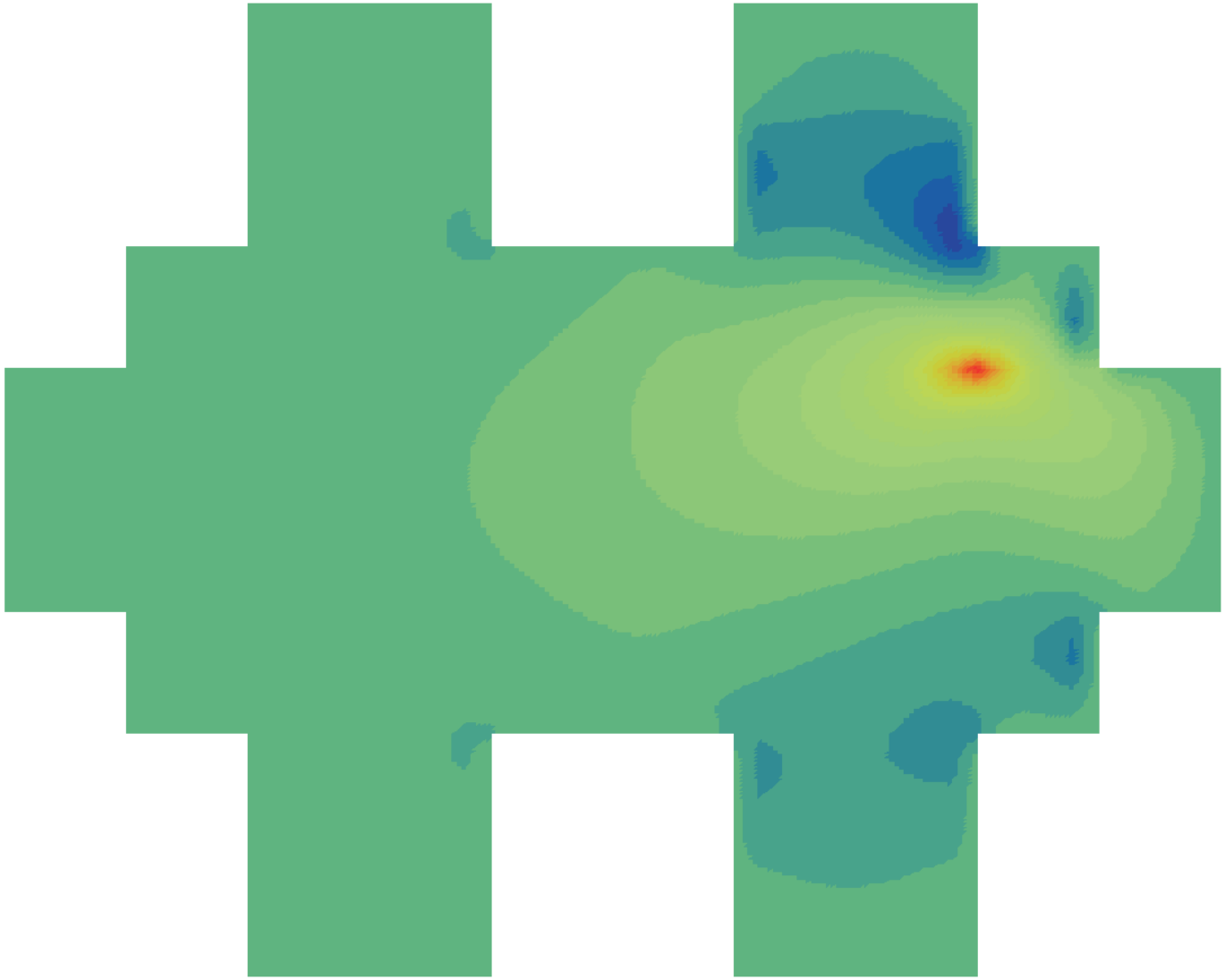
Goal oriented refinement



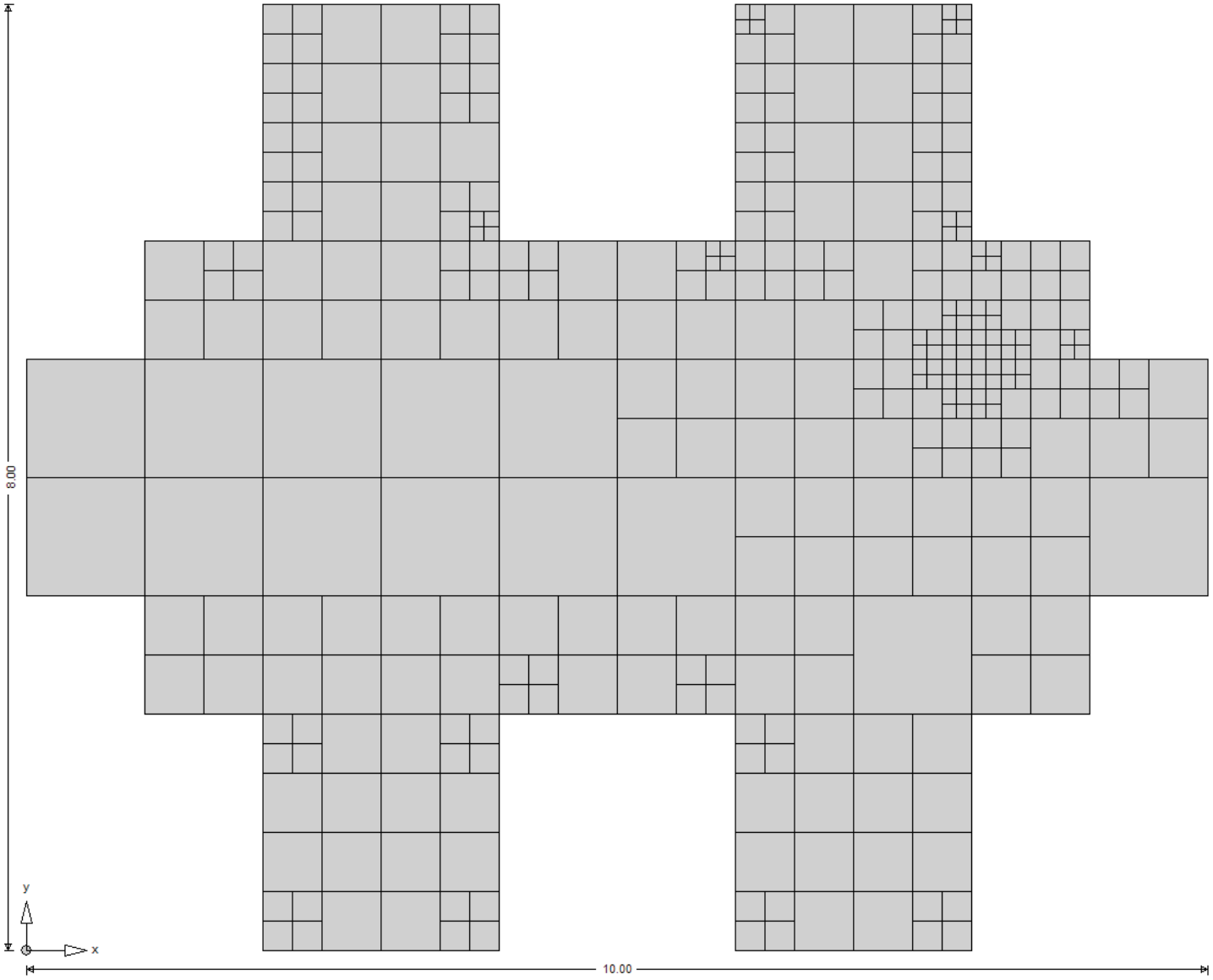
Temperaturfeld



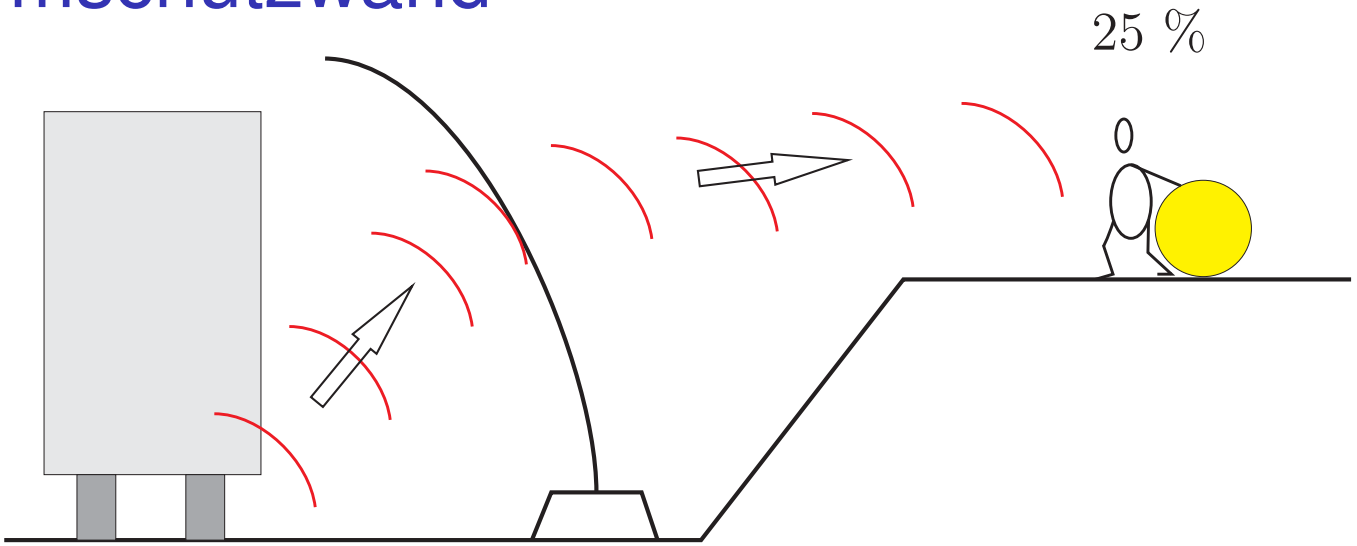
Hot spot

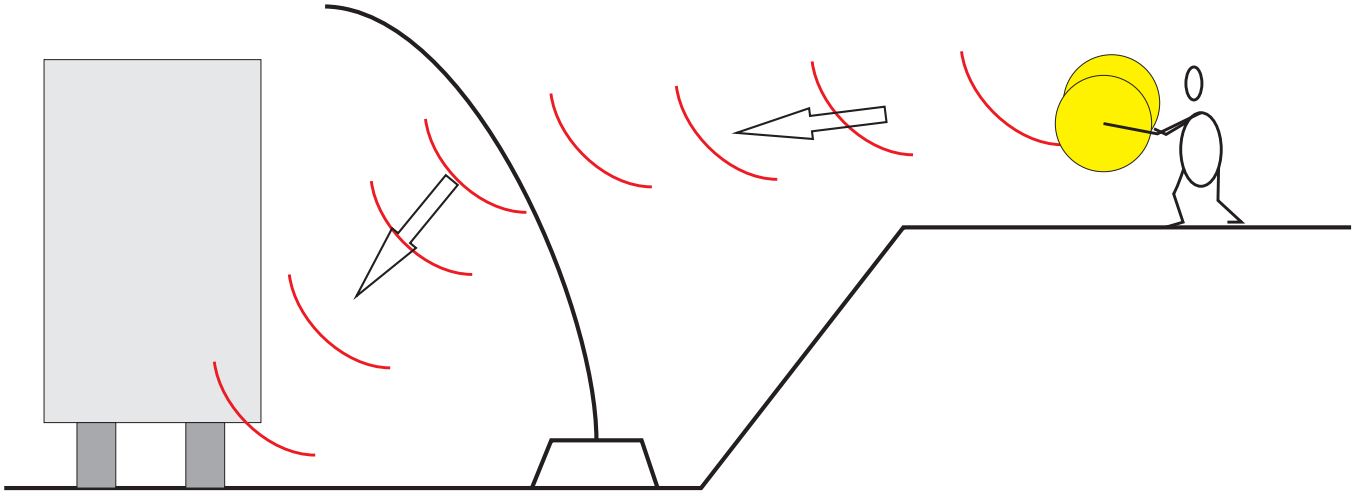


Hot spot



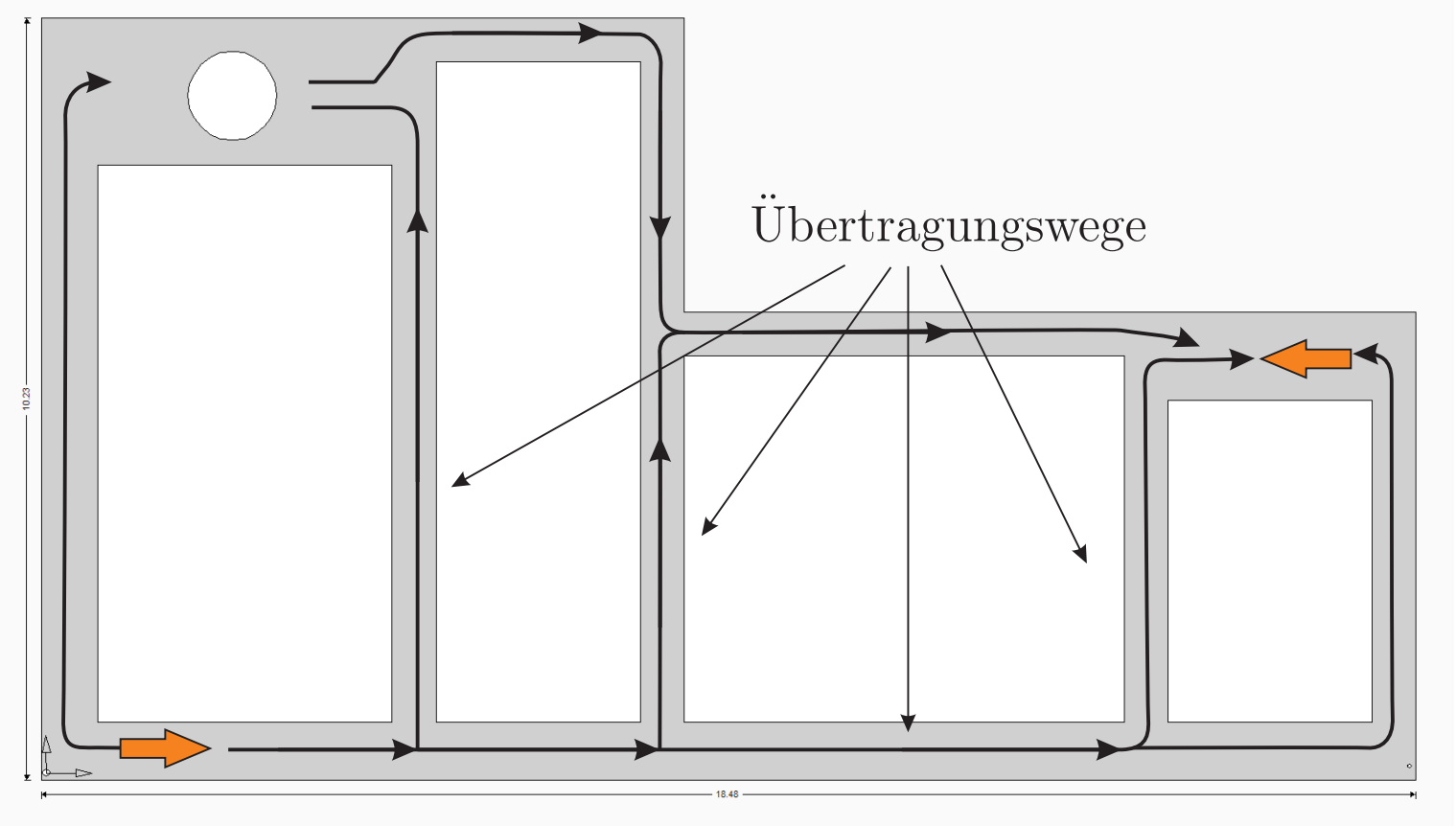
Lärmschutzwand





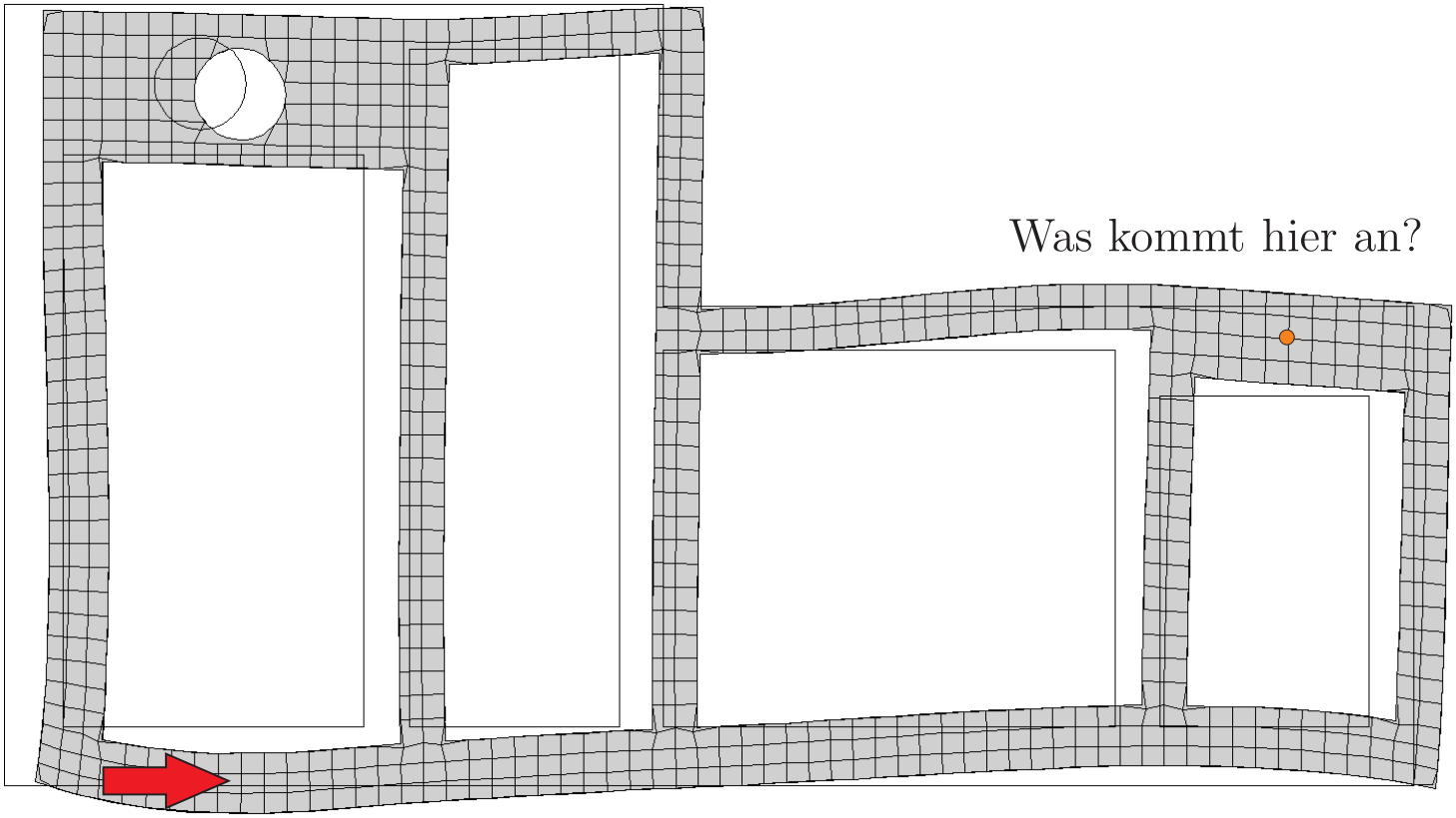
25 %

Kommunikation



Das Echo muss möglichst genau sein!

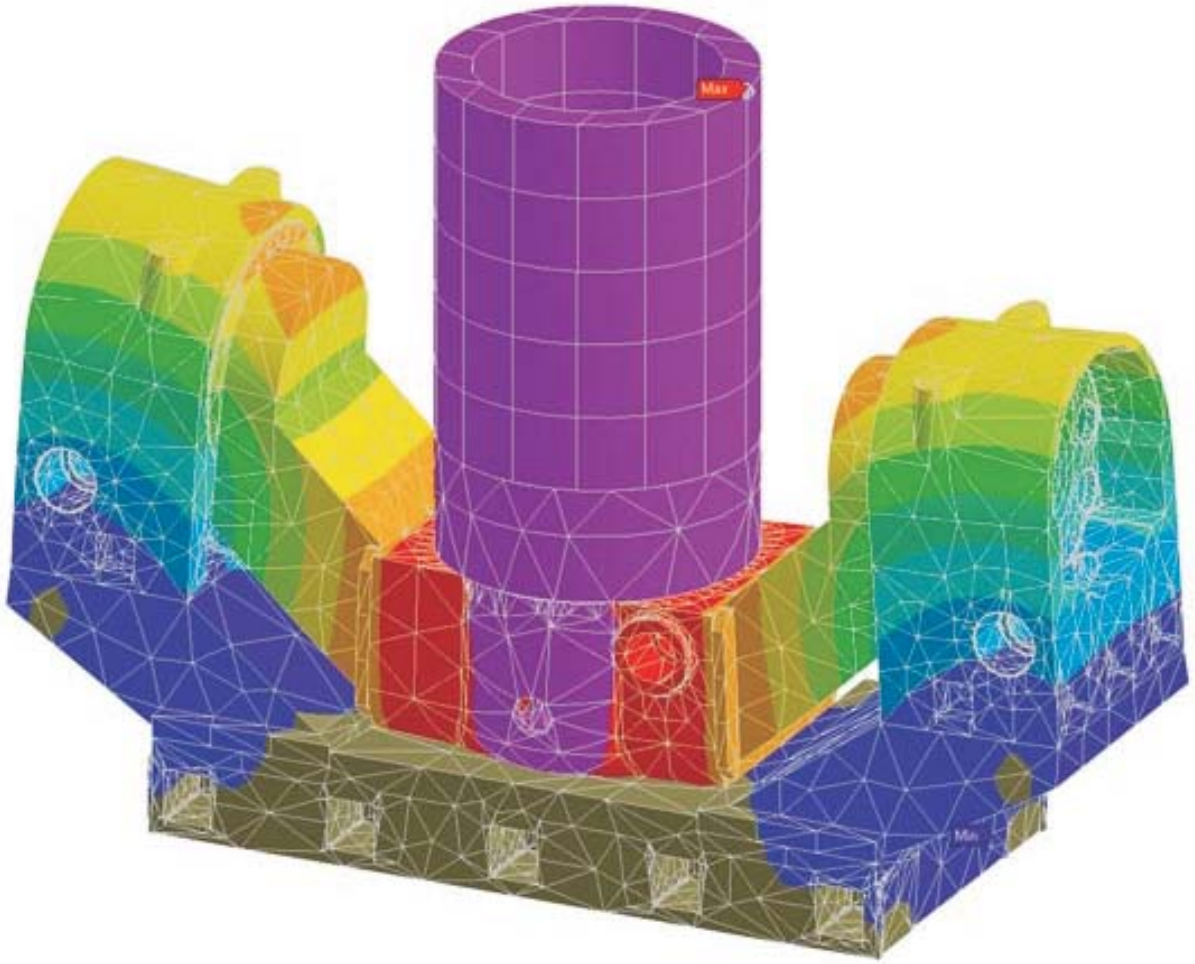
Hot spot



Modellanpassung

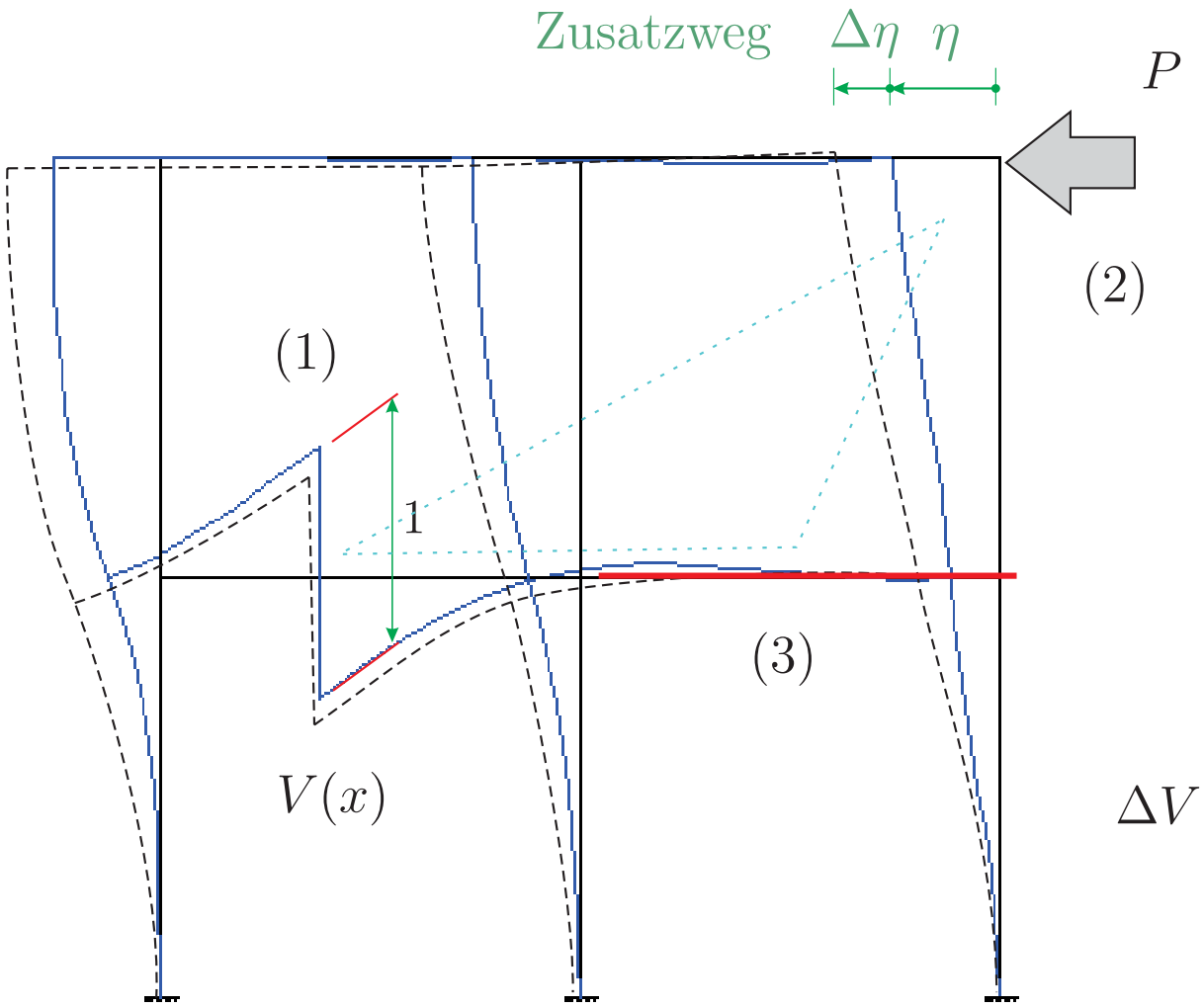
- Änderungen von Steifigkeiten
- Grobes Modell - feines Modell: Abschätzungen
- Optimierung

Wann ist fein fein genug?



http://www.rueckle-gruppe.de/uploads/pics/FEM-Berechnungen.jpg

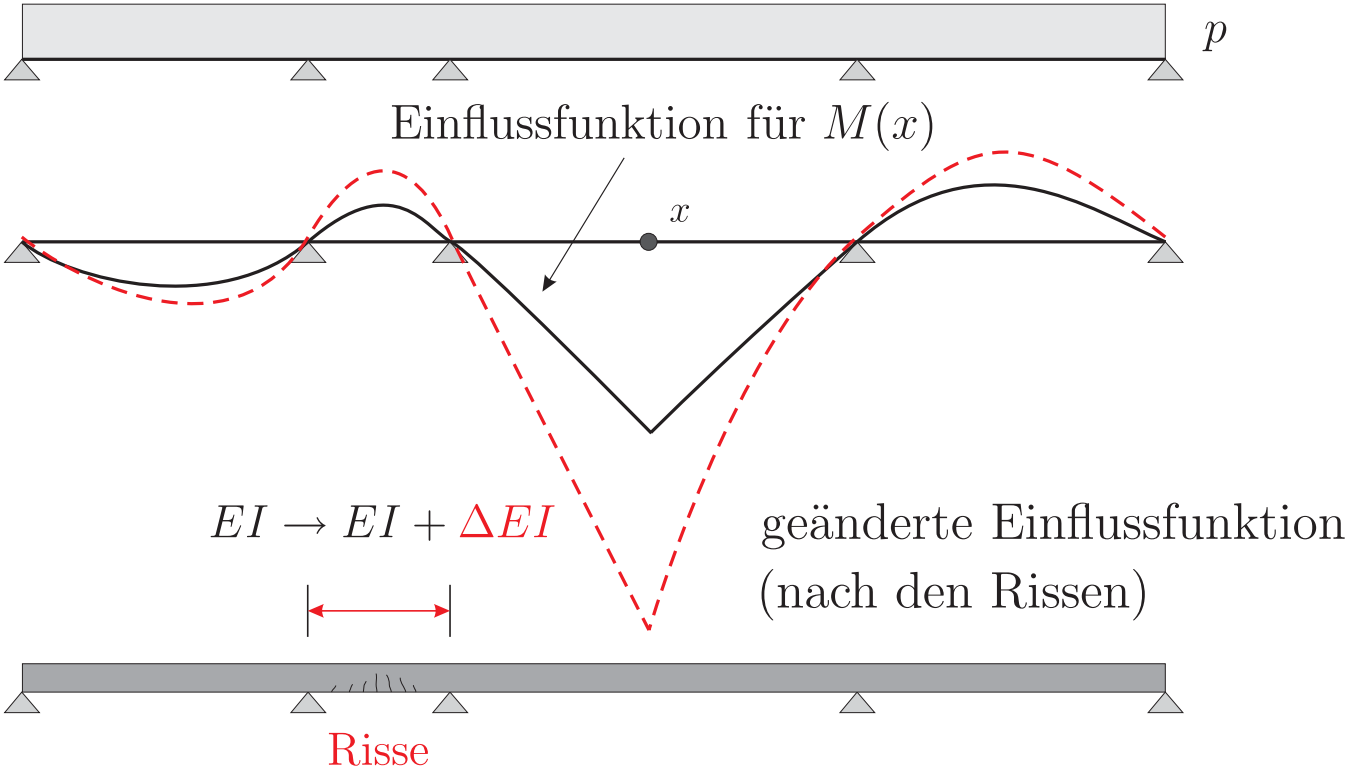
Lokale Änderungen haben globale Effekte



$$\Delta V = P \cdot \Delta\eta$$

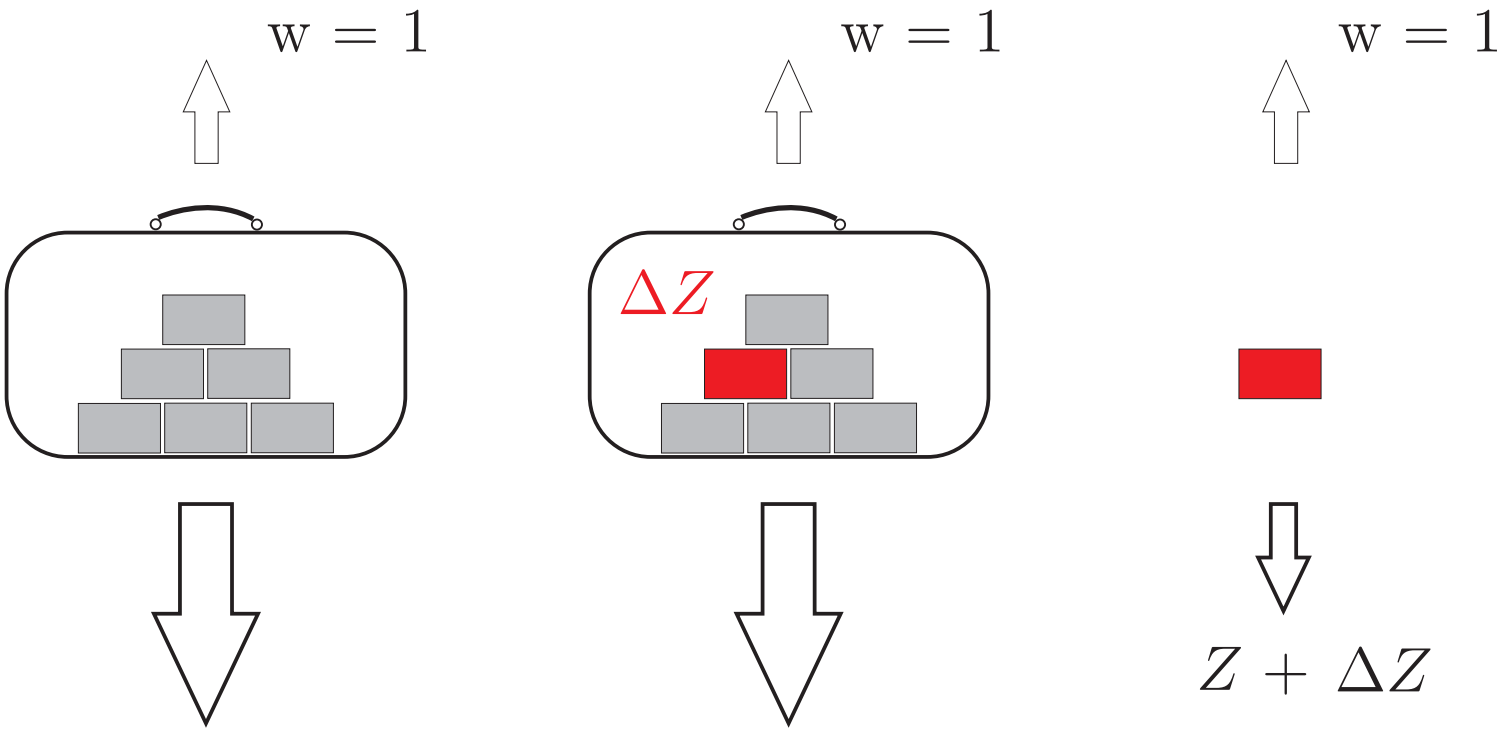
'Dreiecksbeziehung'

Änderungen in den Steifigkeiten



$$M(x) = \int_0^l G_2(y,x)\,p(y)\,dy \qquad M_c(x) = \int_0^l G_2^c(y,x)\,p(y)\,dy$$

Der 'Trick'



G $\qquad$ G_{neu} $\qquad$ $\Delta G = \Delta Z \cdot 1$

$$\Delta G = G_{neu} - G = \Delta Z$$

Geänderte Einflussfunktion $G \to G_c$

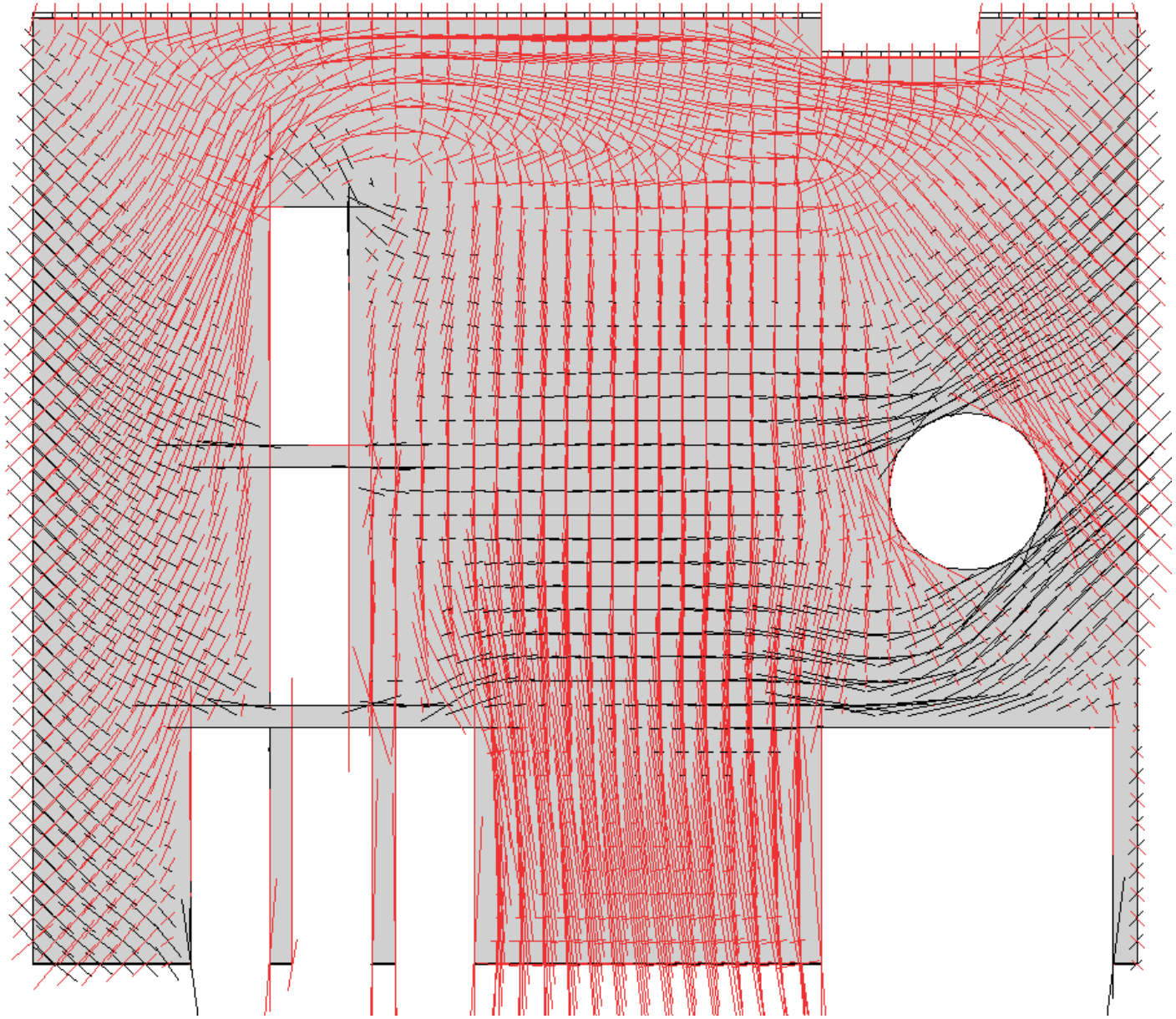
Globale Analyse (Der ganze Koffer)

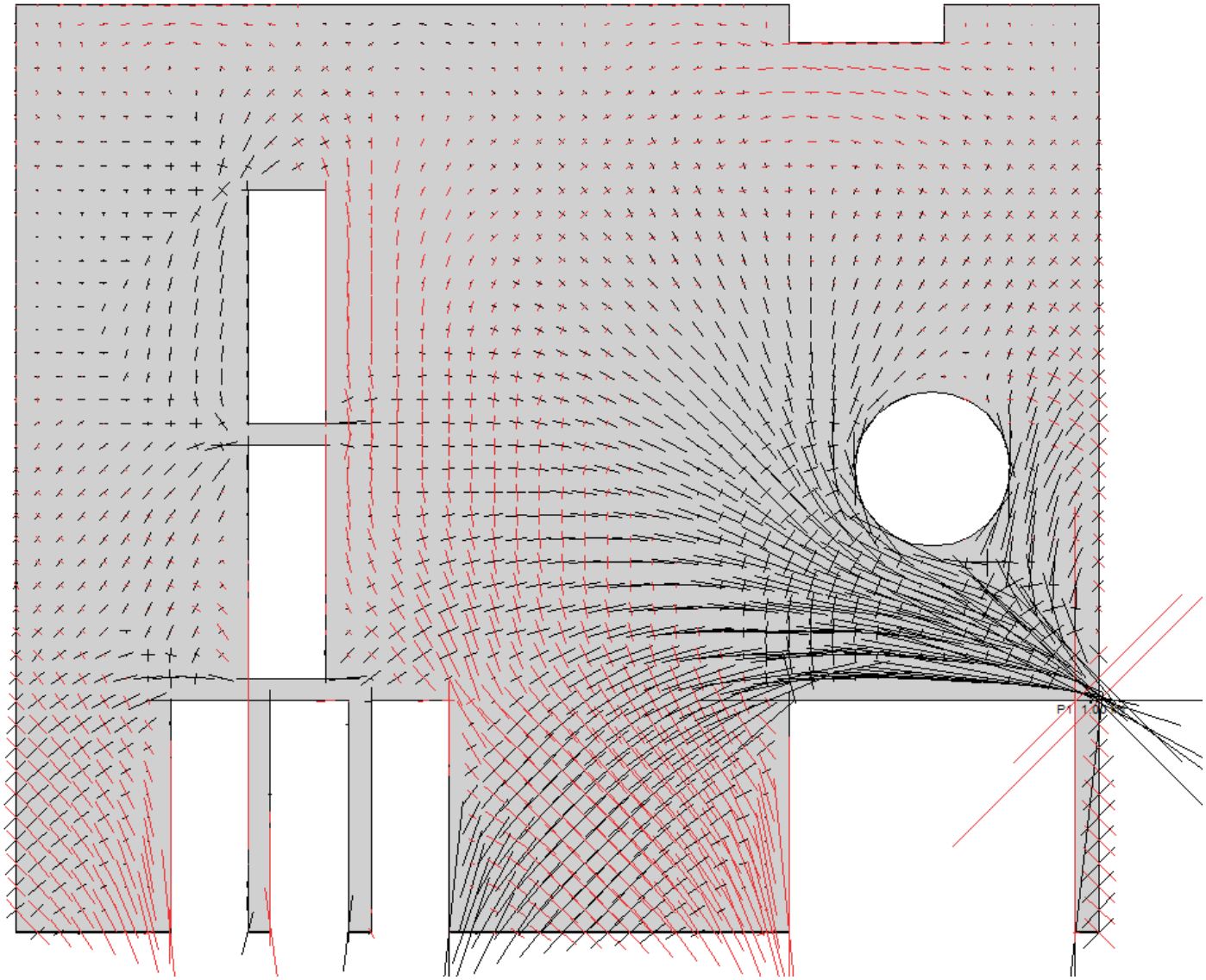
$$u_c - u = \int_0^l [G_c(y,x) - G(y,x)]\, p(y)\, dy \qquad \text{(Betti)}$$

Lokale Analyse $[x_1, x_2]$ (nur der Ziegelstein)

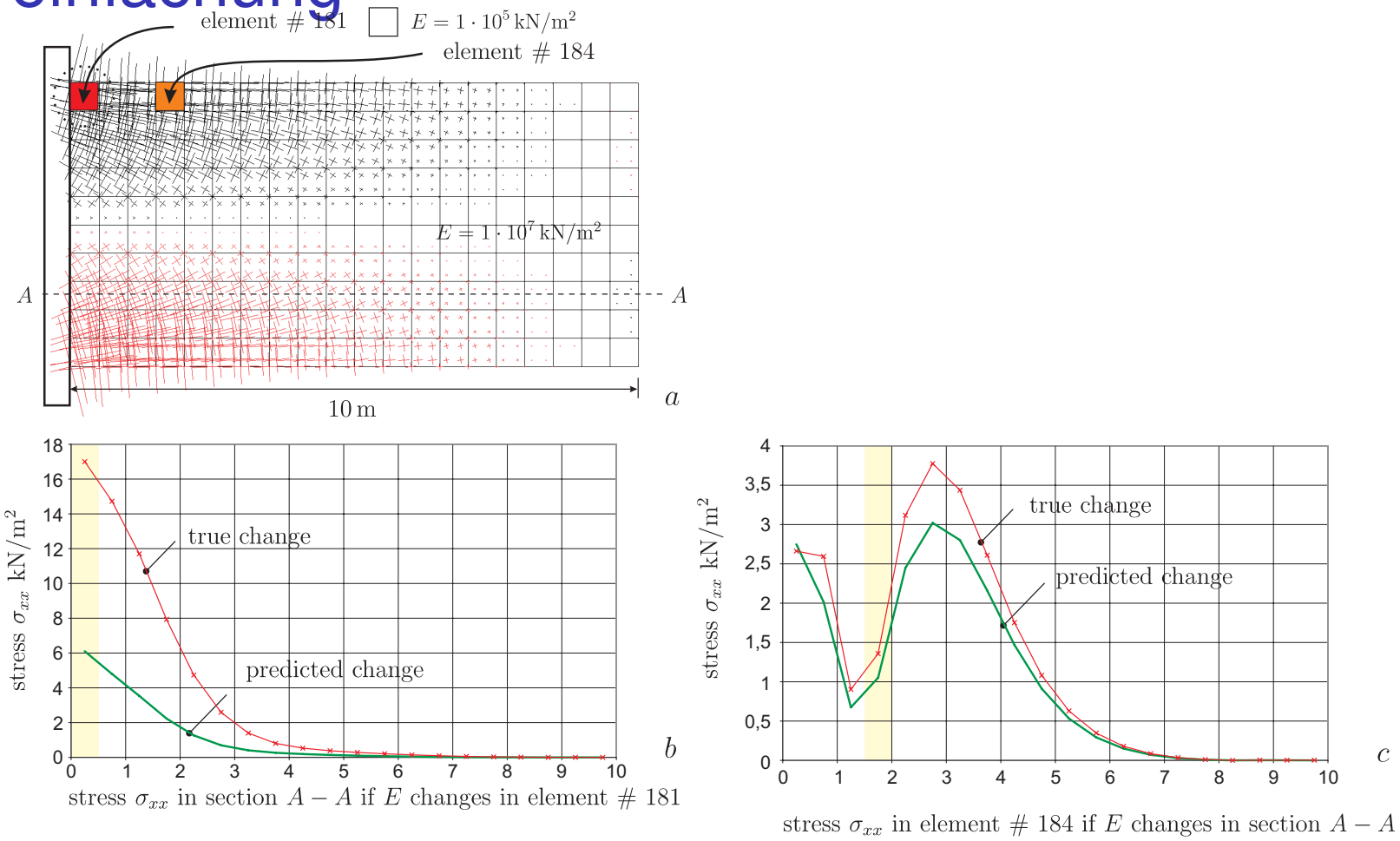
$$u_c - u = \int_{x_1}^{x_2} \Delta EI\, w_c''\, G''\, dy = \frac{\Delta EI}{EI} \int_{x_1}^{x_2} \frac{M_c\, M_G}{EI}\, dy$$

Die lokale Analyse misst die Änderung über die virtuelle innere Energie.



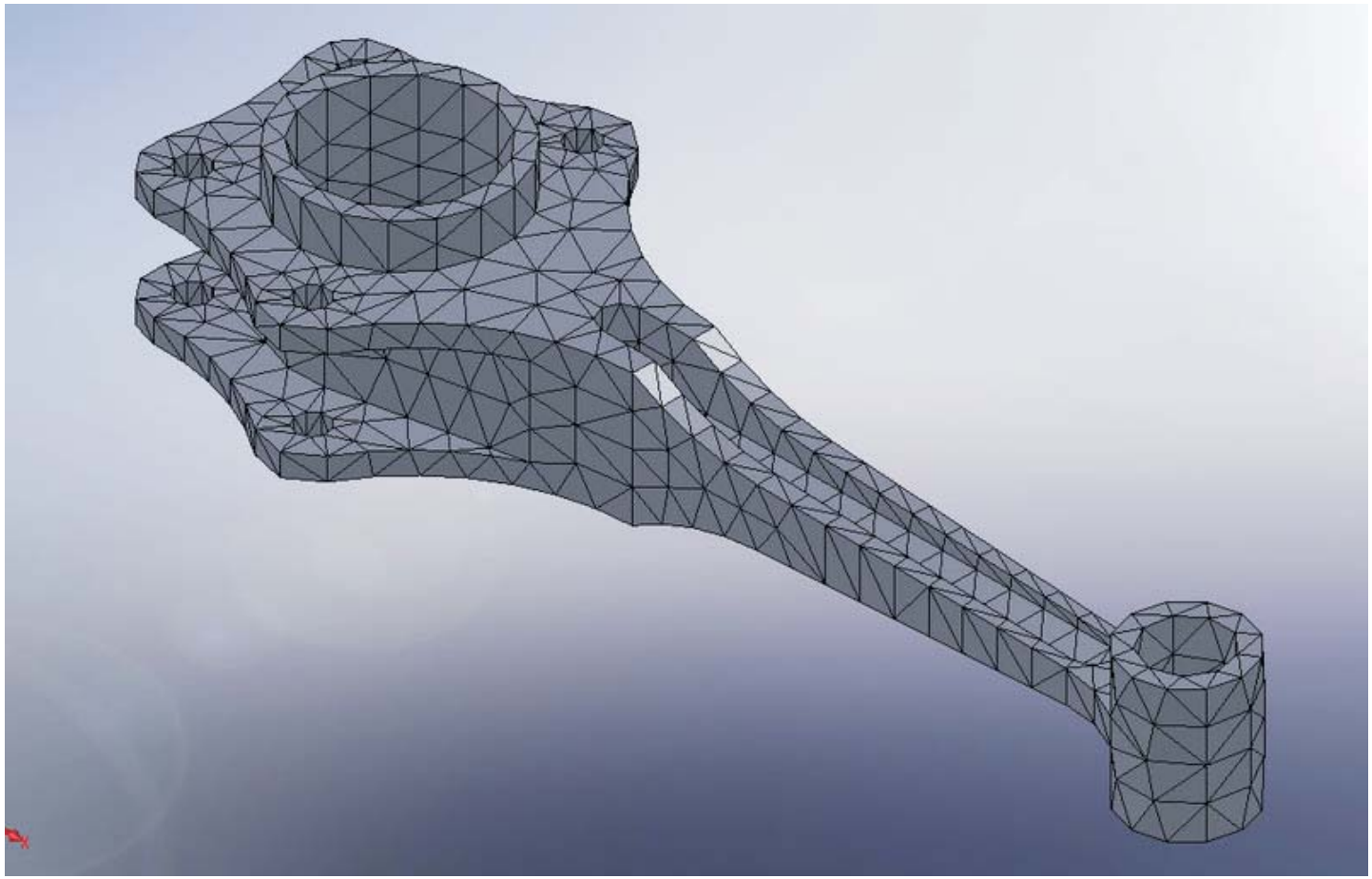

P1

Vereinfachung



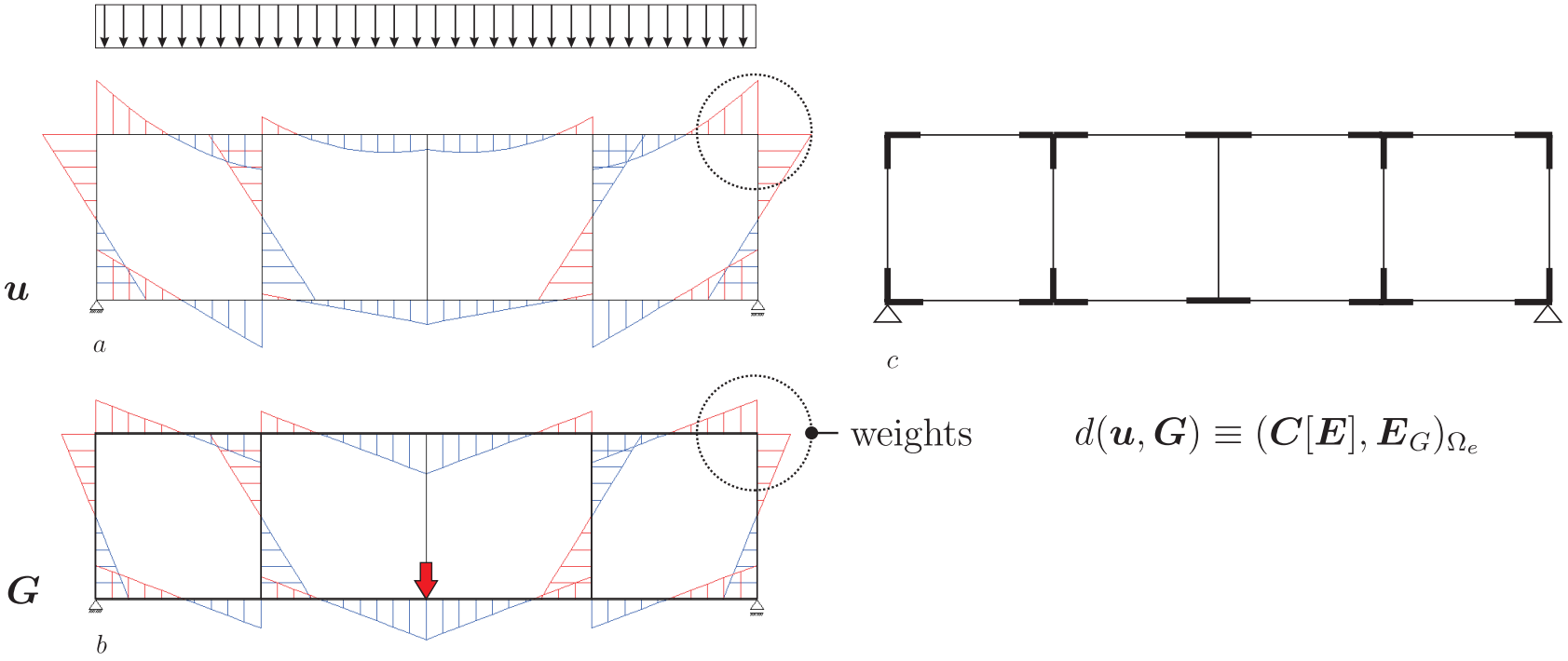
$$u_c - u = -d(u, G_c) = -d(u, G) - d(u, G_c - G)$$
$$\simeq -d(u, G)$$

Retrofitting structures



http://gabijack.com/wp-content/uploads/2008/12/cosmos2.jpg

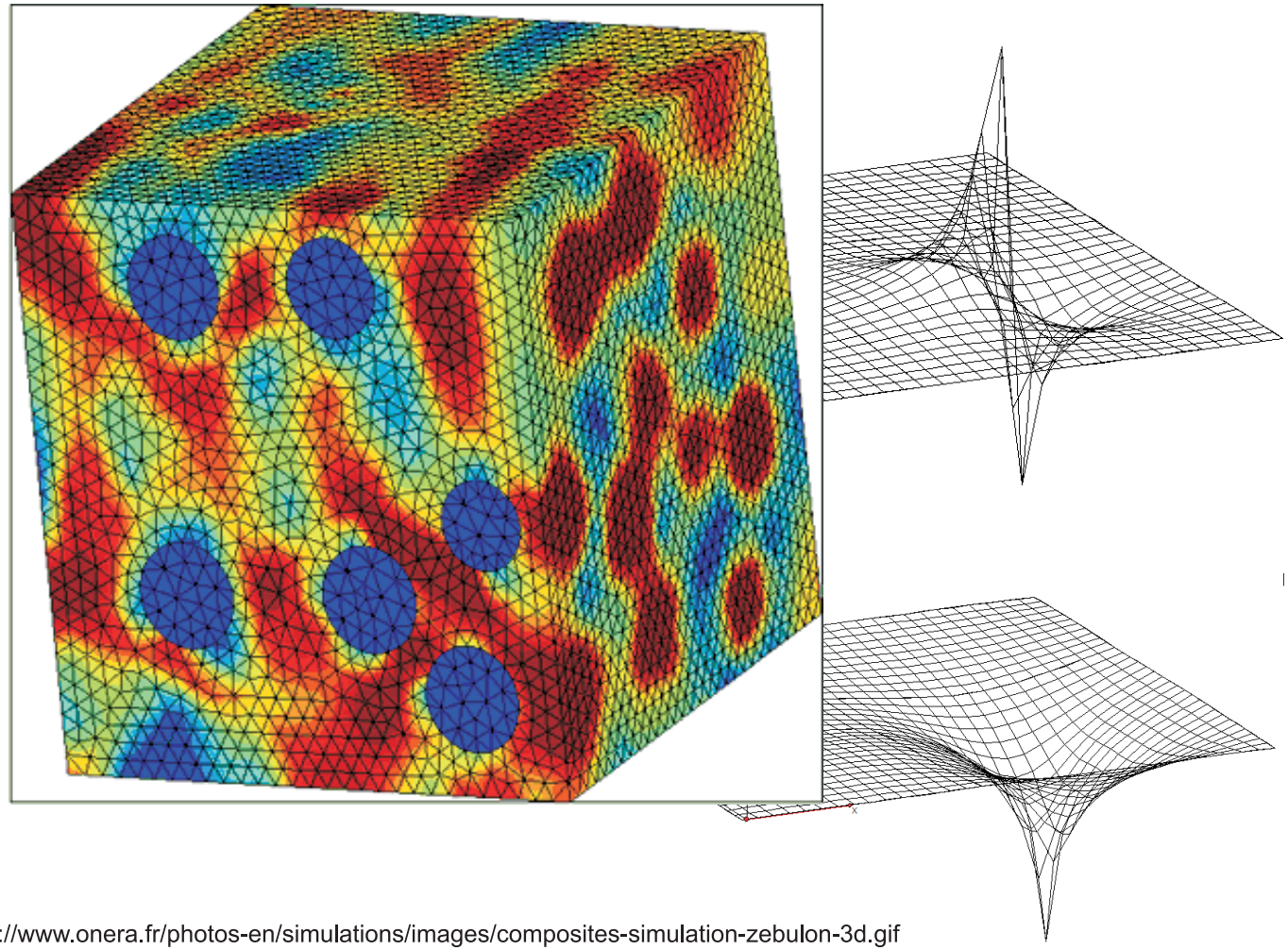
Retrofitting structures



$$d(\boldsymbol{u}, \boldsymbol{G}) \equiv (\boldsymbol{C}[\boldsymbol{E}], \boldsymbol{E}_G)_{\Omega_e}$$

$$u_c^h - u_h \simeq -d(G_h, u_h)$$

Inklusionen



http://www.onera.fr/photos-en/simulations/images/composites-simulation-zebulon-3d.gif

Zusammenfassung

- Mechanik ist angewandte Kinematik
- Zu jeder Zahl gehört eine Bewegung, eine Einflussfunktion
- FE = eingeschränkte Kinematik = ungenaue Einflussfunktionen = ungenaue Ergebnisse
- Goal oriented refinement = gezielte Anpassung des Netz
- Sensitvitätsanalyse mit Einflussfunktionen

Literatur

Bangerth W, Rannacher R. (2003) Adaptive Finite Element Methods for Differential Equations. Birkhäuser Verlag

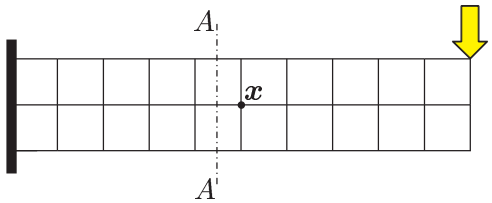
Braack M, Ern A (2003) “A posteriori control of modeling errors and discretization errors”, Multiscale Model. Simul. 1: 221–238

Ern A, Guermond J-L (2004) Theory and Practice of Finite Elements, Springer-Verlag

Hartmann F, Katz C. (2007) Structural Analysis with Finite Elements, 2nd ed., Springer-Verlag

www.winfem.de (Diplomarbeiten, Doktorarbeiten, Programme zu dem Thema)

Nichtlineares Problem



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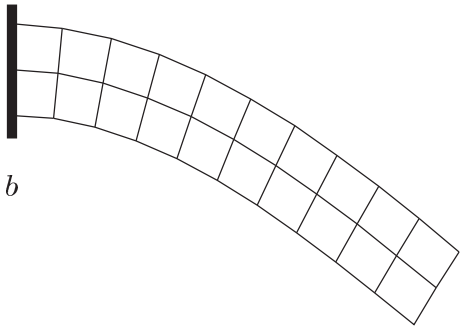
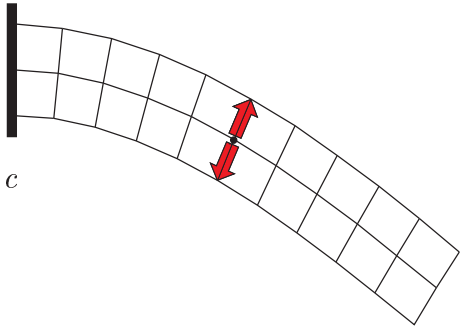


b

Duales Problem für $\sigma_{yy}(\boldsymbol{x})$



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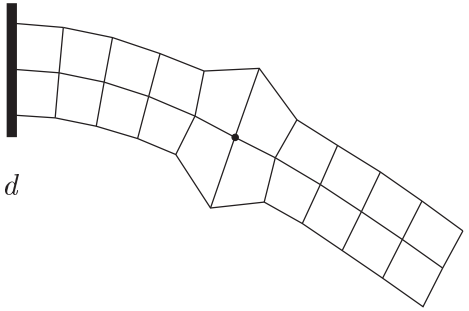
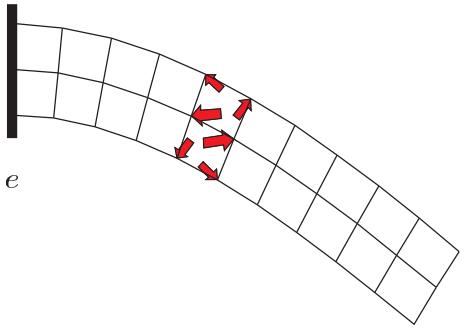


d

Duales Problem für $\int_0^l \sigma_{xy}(\boldsymbol{x})\, ds$



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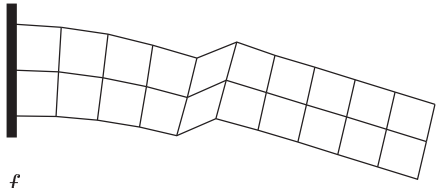

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